Univerzitet u Nišu
Fakultet zaštite na radu u Nišu

ZADACI ZA PRIPREMU KOLOKVIJUMA
I PISANOG DELA ISPITA IZ PREDMETA
OSNOVI MAŠINSTVA

Predmetni nastavnik: dr Darko Mihajlov, vanr. prof.

- Nerecenzirani materijal -

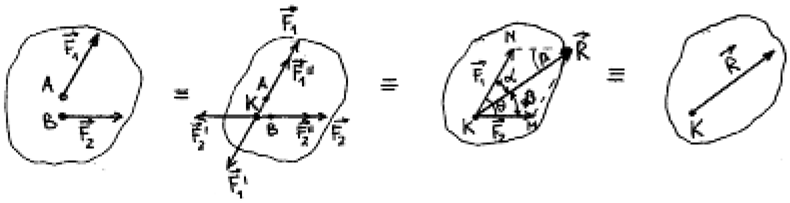
Niš, 2023.

STATIKA

REZULTANTA RAVANSKIH SILA

ZADATAK BR. 1

Odrediti rezultantu komplanarnih sila intenziteta $F_1 = 5$ [kN] i $F_2 = 3$ [kN] koje napadaju kruto telo, tako da napadne linije sila grade među sobom ugao od 60°.



$$\vec{F}_1 = \vec{F}_1'' = -\vec{F}_1'$$
$$\vec{F}_2 = \vec{F}_2'' = -\vec{F}_2'$$

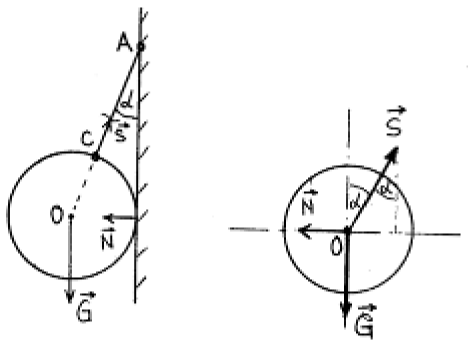
$$\vec{R} = \vec{F}_1 + \vec{F}_2$$
$$\vec{R} \cdot \vec{R} = (\vec{F}_1 + \vec{F}_2) \cdot (\vec{F}_1 + \vec{F}_2)$$
$$R^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta$$
$$R^2 = 5^2 + 3^2 + 2 \cdot 5 \cdot 3 \cdot \cos 60°$$
$$R^2 = 49 \Rightarrow R = 7 \text{ [kN]}$$

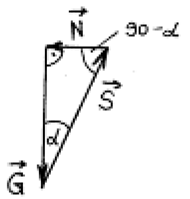
$$\theta = \alpha + \beta$$
$$\frac{F_1}{\sin\beta} = \frac{F_2}{\sin\alpha} = \frac{R}{\sin\theta}$$
$$\sin\alpha = \frac{F_2}{R}\sin\theta = \frac{3}{7} \cdot \frac{\sqrt{3}}{2} \approx 0{,}3711 \Rightarrow \alpha \approx 22°$$
$$\beta = \theta - \alpha \Rightarrow \beta \approx 38°$$

RAVANSKI SISTEM SUČELJNIH SILA

ZADATAK BR. 2

O vertikalni glatki zid $\overline{AB}$ oslonjena je kugla 0, obešena o konac AC. Ugao koji konac zatvara sa zidom je α, a težina kugle je G. Odrediti silu S u koncu i pritisak N kugle na zid.





$$\frac{S}{\sin 90^\circ} = \frac{N}{\sin\alpha} = \frac{G}{\sin(90-\alpha)}$$

$$S = G\,\frac{\sin 90}{\cos\alpha} = \frac{G}{\cos\alpha}$$

$$N = G\,\frac{\sin\alpha}{\cos\alpha} = G\,tg\alpha$$

II način:

$$\vec{R} = 0$$

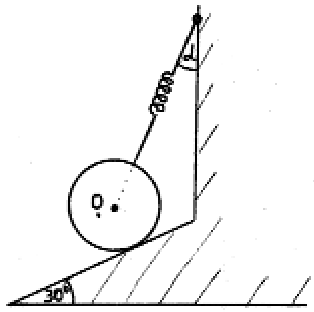
$$\vec{R} = R_x\,\vec{i} + R_y\,\vec{j}$$

$$R_x = \sum X_i = 0 \Rightarrow S\cos(90-\alpha) - N = 0 \Rightarrow S = \frac{N}{\sin\alpha}$$

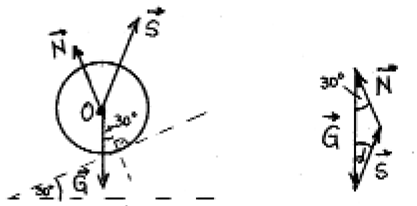
$$R_y = \sum Y_i = 0 \Rightarrow S\sin(90-\alpha) - G = 0 \Rightarrow S = \frac{G}{\cos\alpha} \Rightarrow N = G\,tg\alpha$$

ZADATAK BR. 3

Pomoću konca održava se na glatkoj strmoj ravni kugla O, težine 20 [kN]. Konac je vezan za opružnu vagu, koja pokazuje silu od 10 [kN]. Strma ravan zatvara sa horizontalom ugao od 30°.
Odrediti ugao α koji konac zatvara sa vertikalom i pritisak N kugle na strmu ravan.



$G = 20$ [kN]
$S = 10$ [kN]
$\varphi = 30°$
$\alpha = ?$
$N = ?$



$$\frac{S}{\sin 30°} = \frac{N}{\sin\alpha} = \frac{G}{\sin(180° - 30° - \alpha)}$$

$$\sin(150° - \alpha) = \frac{G}{S}\sin 30°$$

$$\sin 150° \cos\alpha - \cos 150° \sin\alpha = \frac{G}{S}\sin 30°$$

$$\frac{1}{2}\cos\alpha + \frac{\sqrt{3}}{2}\sin\alpha = \frac{20}{10}\cdot\frac{1}{2}$$

$$\cos\alpha + \sqrt{3}\sin\alpha = 2$$

$$-\cos\alpha + 2 = \sqrt{3}\sin\alpha \;/^2$$

$$\cos^2\alpha + 4 - 4\cos\alpha = 3\sin^2\alpha \;; \quad \sin^2\alpha = 1 - \cos^2\alpha$$

$$\cos^2\alpha + 4 - 4\cos\alpha = 3 - 3\cos^2\alpha$$

$$4\cos^2\alpha - 4\cos\alpha + 1 = 0$$

$$\cos\alpha_{1,2} = \frac{4 \pm \sqrt{16 - 4\cdot 4\cdot 1}}{8} = \frac{1}{2}$$

$$\cos\alpha = \frac{1}{2} \Rightarrow \underline{\alpha = 60°}$$

$$N = S\,\frac{\sin 60°}{\sin 30°} \Rightarrow \underline{N = 10\sqrt{3}\ [kN]}$$

ZADATAK BR. 4

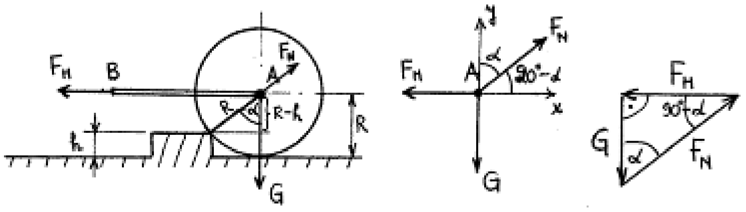
Kojom horizontalnom silom F_H treba vući držalju valjka da bi prešao prag visine $h = 8$ [cm]. Poluprečnik valjka $R = 60$ [cm], a njegova težina $G = 2$ [kN].

$F_H = ?$

$G = 2$ [kN]

$h = 8$ [cm]

$R = 60$ [cm]



$$\frac{F_H}{\sin\alpha} = \frac{F_N}{\sin 90^\circ} = \frac{G}{\sin(90^\circ-\alpha)}$$

$$F_H = G\frac{\sin\alpha}{\cos\alpha} = G\,tg\alpha$$

$$\cos\alpha = \frac{R-h}{R}$$

$$tg\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{\sin\alpha}{\frac{R-h}{R}} = \frac{R\sin\alpha}{R-h} = \frac{R\sqrt{1-\left(\frac{R-h}{R}\right)^2}}{R-h} = \frac{\sqrt{R^2-R^2+2Rh+h^2}}{R-h} = \frac{\sqrt{2Rh+h^2}}{R-h}$$

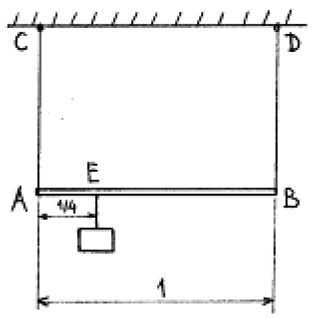
$$tg\alpha = \frac{\sqrt{2\cdot 60\cdot 8-8^2}}{60-8} = \frac{\sqrt{896}}{52} = 0{,}5756 \Rightarrow \alpha = 29{,}926^\circ$$

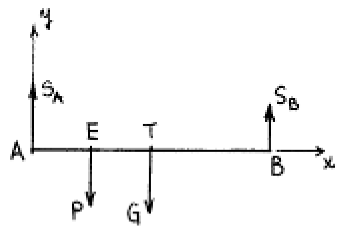
$$F_H = G\,tg\alpha = 2\cdot 0{,}5756 \rightarrow F_H = 1{,}1522\ [kN]$$

RAVANSKI SISTEM PARALELNIH SILA

ZADATAK BR. 5

Homogeni štap AB, dužine 1[m], težine 2[kN], obešen je horizontalno o dva kanapa AC i BD. Za štap je obešen u tački E teret P=12 [kN]. Odstojanje AE = $\frac{1}{4}$ [m]. Odrediti sile u kanapima.





1) $\sum X_i = 0$

2) $\sum Y_i = S_A - P - G + S_B = 0$

3) $\sum M_A = -P\frac{1}{4} - G\frac{1}{2} + S_B \cdot 1 = 0$

$$S_B = \frac{P}{4} + \frac{G}{2}$$

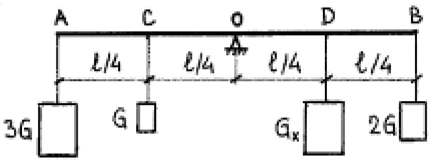
$$S_B = 3 + 1 \Rightarrow \underline{S_B = 4\ [kN]}$$

2) $\Rightarrow$ $S_A = P + G - S_B$

$$S_A = 12 + 2 - 4 \Rightarrow \underline{S_A = 10\ [kN]}$$

ZADATAK BR. 6

O tačke A i B ravnokrake dvokrake poluge AB obešeni su tereti težina 3G i 2G, a o tačku C ($\overline{AC}=\overline{CO}$) teret težine G. Kolikim se teretom mora delovati u tački D ($\overline{OD}=\overline{DB}$) da bi poluga bila u ravnoteži?



$\overline{AO}=\overline{OB}$; $\overline{AC}=\overline{CO}$; $\overline{OD}=\overline{DB}$

$\Sigma M_o = 0$

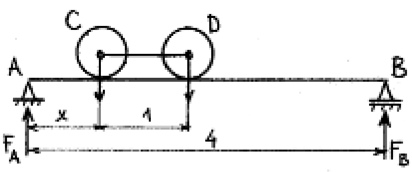
$$3G\frac{\ell}{2} + G\frac{\ell}{4} = 2G\frac{\ell}{2} + G_x\cdot\frac{\ell}{4} \quad / \cdot \frac{4}{\ell}$$

$$G_x = 4G\left(\frac{3}{2}+\frac{1}{4}-1\right) = 3G$$

ZADATAK BR. 7

Duž horizontalne grede na dva oslonca mogu se pomerati dva tereta, jedan C = 200 [kN], drugi D = 100 [kN]. Raspon grede je L = 4 [m], $\overline{CD}$ = 1 [m]. Na kom rastojanju x od oslonca A mora da se nalazi teret C da bi otpor oslonca A bio dva puta veći od otpora oslonca B? Uticaj težine grede zanemariti.

C = 200 [kN]
D = 100 [kN]
$\overline{AB}$ = 4 [m]
$\overline{CD}$ = 1 [m]
x = ?
$F_A = 2F_B$



$$\Sigma Y_i = F_A + F_B - C - D = 0$$

$$F_A = 2F_B \Rightarrow 3F_B = C + D, \quad F_B = \frac{C+D}{3}$$

$$F_B = 100\,[kN] \; ; \; F_A = 200\,[kN]$$

$$\Sigma M_A = F_B\cdot\overline{AB} - D(x+\overline{CD}) - Cx = 0$$

$$x = \frac{F_B\cdot\overline{AB} - D\cdot\overline{CD}}{C+D} = \frac{100\cdot 4 - 100\cdot 1}{200+100} = \frac{300}{300}$$

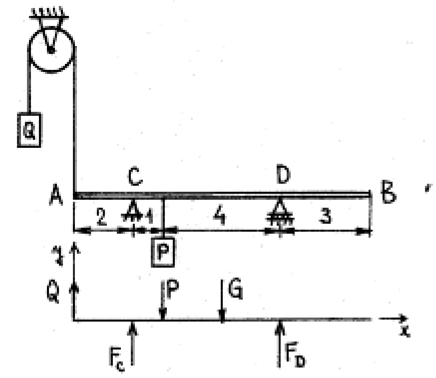
$$x = 1\,[m]$$

ZADATAK BR. 8

Homogena prizmatična greda AB, dužine 10m, težine 200 kN, oslonjena je u tačkama C i D. Rastojanje oslonca C od kraja A grede jednako je 2 m, rastojanje oslonca D od kraja B grede jednako je 3 m. Na rastojanju 3 m od kraja A grede obešen je teret P = 800 kN. Za kraj A grede vezano je uže koje je prebačeno preko kotura i zategnuto teretom Q = 300 kN.
Odrediti otpore oslonaca.

$\overline{AB} = 10$ [m] $P = 800$ [kN]
$\overline{AC} = 2$ [m] $Q = 300$ [kN]
$\overline{DB} = 3$ [m] $G = 200$ [kN]
$\overline{AP} = 3$ [m]

$F_c, F_D = ?$



1) $\Sigma Y_i = 0$: $Q + F_c - P - G + F_D = 0$

2) $\Sigma M_A = 0$: $F_c \cdot 2 - P \cdot 3 - G \cdot 5 + F_D \cdot 7 = 0$

1) $\Rightarrow F_c + F_D = P + G - Q \quad \Rightarrow \quad F_c + F_D = 700 \quad / \cdot (-2)$

2) $\Rightarrow 2F_c + 7F_D = 3P + 5G \quad \Rightarrow \quad 2F_c + 7F_D = 3400$

$+ \Rightarrow 5F_D = 2000$

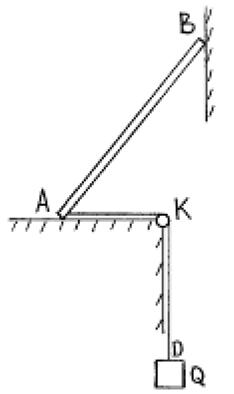
$F_D = 400$ [kN]

$F_c = 300$ [kN]

RAVANSKI SISTEM PROIZVOLJNIH SILA

ZADATAK BR. 9

Homogeni štap AB, dužine l, težine G, oslanja se na vertikalni zid krajem B i na pod krajem A. Za kraj A vezano je lako uže, koje je prebačeno preko malog kotura K, o čiji drugi kraj D visi teret Q.
Odrediti ugao koji štap AB gradi sa horizontalnim podom ako je $G/Q = 2\sqrt{3}$.
Odrediti reakcije u tačkama oslanjanja ako je $Q = 20$ [kN].
Trenje zanemariti.



$Q = 20$ [kN] , $G = Q \cdot 2\sqrt{3} = 40\sqrt{3}$ [kN]

$\sum X_i = 0: \quad Q - F_B = 0 \Rightarrow F_B = Q = 20$ [kN]

$\sum Y_i = 0: \quad F_A - G = 0 \Rightarrow F_A = G = 40\sqrt{3}$ [kN]

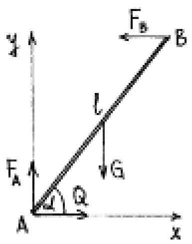
$\sum M_A = 0: \quad F_B \cdot l \sin\alpha - G \frac{l}{2} \cos\alpha = 0$

$$F_B \sin\alpha = \frac{1}{2} G \cos\alpha$$

$$\frac{\sin\alpha}{\cos\alpha} = \frac{G}{2F_B}$$

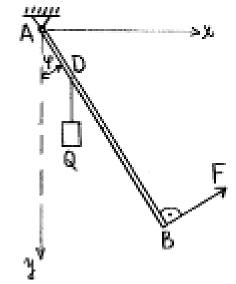
$$tg\,\alpha = \frac{40\sqrt{3}}{2 \cdot 20} = \sqrt{3}$$

$$\alpha = arc\,tg\sqrt{3} = 60^\circ$$



ZADATAK BR. 10

Homogeni štap AB, dužine L i težine G, zglobno je vezan u tački A.
U tački D ($\overline{AD}=L/4$) obešen je teret Q, a na kraj B štapa dejstvuje koncentrisana sila F upravno na osu štapa.
Za ravnotežni položaj štapa odrediti ugao koji osa štapa gradi sa vertikalom i silu u zglobu, ako je $G=50$ kN, $Q=4F=3G$, $L=0{,}8$ m.

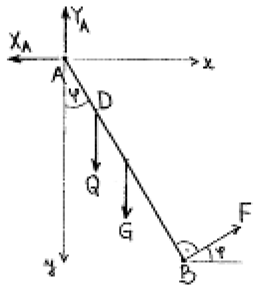


$$\sum M_A=0:\quad F\cdot\cancel{L}-G\frac{\cancel{L}}{2}\sin\varphi-Q\frac{\cancel{L}}{4}\sin\varphi=0\;/\cdot 4$$

$$4F=(2G+Q)\sin\varphi$$

$$\sin\varphi=\frac{4F}{2G+Q}=\frac{3G}{2G+3G}=\frac{3}{5}$$

$$\cos\varphi=\sqrt{1-\sin^2\varphi}=\frac{4}{5}$$



$$\sum X_i=0:\quad X_A-F\cos\varphi=0\;\Rightarrow\;X_A=F\cos\varphi=\frac{4}{5}F=\frac{3}{5}G$$

$$X_A=30\ [\text{kN}]$$

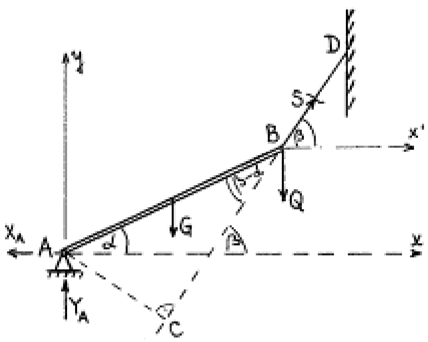
$$\sum Y_i=0:\quad -Y_A+Q+G-F\sin\varphi=0$$

$$Y_A=Q+G-F\sin\varphi=3G+G-\frac{9}{20}G=\frac{71}{20}G$$

$$Y_A=177{,}5\ [\text{kN}]$$

ZADATAK BR. 11

Homogeni štap $\overline{AB}$, dužine L, težine G, vezan je krajem A zglobom za pod. Drugi njegov kraj B vezan je užetom za zid. O kraj B štapa visi teret Q. Odrediti sile veza ako štap gradi ugao α sa horizontalom, a uže ugao β.



1) $\Sigma X_i = 0: \ -X_A + S\cos\beta = 0 \Rightarrow X_A = S\cos\beta$

2) $\Sigma Y_i = 0: \ Y_A - G - Q + S\sin\beta = 0 \Rightarrow Y_A = G + Q - S\sin\beta$

3) $\Sigma M_A = 0: \ -G\frac{L}{2}\cos\alpha - Q \cdot L\cos\alpha + S \cdot L\sin(\beta-\alpha) = 0$

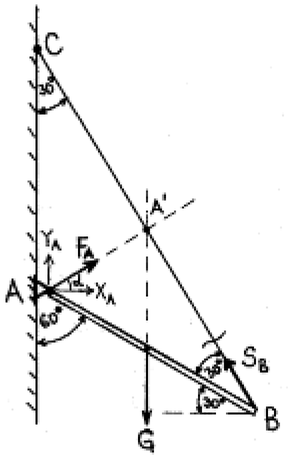
$$S = \frac{(G+2Q)\cos\alpha}{2\sin(\beta-\alpha)}$$

1) $\Rightarrow$ $$X_A = \frac{(G+2Q)\cos\alpha}{2\sin(\beta-\alpha)}\cdot\cos\beta$$

2) $\Rightarrow$ $$Y_A = G + Q - \frac{(G+2Q)\cos\alpha}{2\sin(\beta-\alpha)}\cdot\sin\beta$$

ZADATAK BR. 12

Homogeni prizmatični štap $\overline{AB}$ vezan je krajem A zglavkasto za zid, a kraj B pridržava uže BC. Osa štapa zatvara sa vertikalom ugao od 60°. Ugao ABC = 30°. Težina štapa iznosi G = 2 [kN]. Odrediti brojnu vrednost, pravac i smer otpora A i silu u užetu S_B.



1) $\sum X_i = 0: \quad X_A - S_B \cos 60° = 0$

2) $\sum Y_i = 0: \quad Y_A - G + S_B \sin 60° = 0$

3) $\sum M_A = 0: \quad S_B \cdot \overline{AA'} - G \frac{\overline{AB}}{2} \sin 60° = 0$

$$S_B = \frac{G}{2} \frac{\overline{AB}}{\overline{AA'}} \sin 60°$$

$$\overline{AA'} = \overline{AB} \cdot \sin 30°$$

$$S_B = \frac{G}{2} \frac{\cancel{\overline{AB}}}{\cancel{\overline{AB}}} \frac{\sin 60°}{\sin 30} = \frac{G}{2} \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$$

$$\underline{S_B = G \frac{\sqrt{3}}{2} = \sqrt{3} \ [kN]}$$

$1) \Rightarrow X_A = S_B \cos 60° = G \frac{\sqrt{3}}{2} \cdot \frac{1}{2} \quad \Rightarrow \quad X_A = G \frac{\sqrt{3}}{4}$

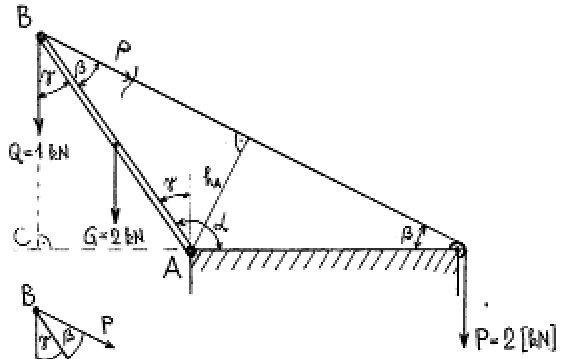
$2) \Rightarrow Y_A = G - S_B \sin 60° = G - G \frac{\sqrt{3}}{2} \frac{\sqrt{3}}{2} \quad \Rightarrow \quad Y_A = \frac{G}{4}$

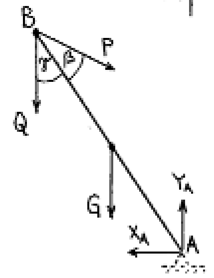
$F_A = \sqrt{X_A^2 + Y_A^2} = \sqrt{\left(G \frac{\sqrt{3}}{4}\right)^2 + \left(\frac{G}{4}\right)^2} = \frac{G}{4} \sqrt{3+1} \quad \Rightarrow \quad \underline{F_A = \frac{1}{2} G = 1 \ [kN]}$

$tg\alpha = \frac{Y_A}{X_A} = \frac{\frac{G}{4}}{\frac{G\sqrt{3}}{4}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \quad \Rightarrow \alpha = arctg\left(\frac{\sqrt{3}}{3}\right), \quad \underline{\alpha = 30°}$

ZADATAK BR. 13

O štap AB, koji se može obrtati oko zgloba A, obešen je u tački B, pomoću konca, teg G, težine 1 kN. Za kraj B štapa AB vezan je drugi konac koji je prebačen preko nepokretnog kotura D i zategnut tegom od 2 kN. Dužina $\overline{AB} = \overline{AD}$. Težina štapa jednaka je 2 kN. Odrediti veličinu ugla $\sphericalangle DAB = \alpha$ pri kojem će štap AB stajati u ravnoteži. Trenje zanemariti.





$$\sum \vec{M}_A = 0 : \quad P \cdot \overline{AB} \sin\beta - Q\,\overline{AB}\sin\gamma - G\,\frac{\overline{AB}}{2}\sin\gamma = 0$$

$$P\sin\beta = \left(Q + \frac{G}{2}\right)\sin\gamma$$

$$2\sin\beta = \left(1 + \frac{2}{2}\right)\sin\gamma$$

$$\sin\beta = \sin\gamma$$

$$\beta = \gamma$$

ΔABC : $\sphericalangle CAB = 90 - \gamma$

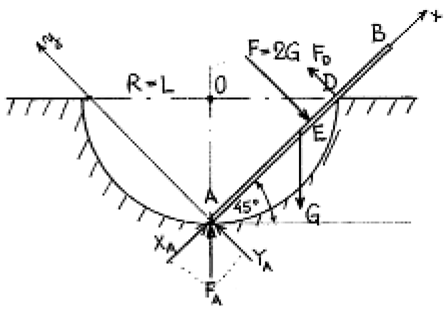
$$\left.\begin{aligned} 90 - \gamma &= 180 - \alpha \\ \alpha + 2\beta &= 180 \end{aligned}\right\} \Rightarrow \left.\begin{aligned} \alpha - \gamma &= 90 \quad /\cdot(-1) \\ \alpha + 2\gamma &= 180 \end{aligned}\right\} + \Rightarrow 3\gamma = 90^\circ \Rightarrow$$

$$\underline{\gamma = \beta = 30^\circ}$$

$$\Rightarrow \underline{\alpha = 120^\circ}$$

ZADATAK BR. 14

Homogeni štap $\overline{AB}$, dužine $2L$, težine G, postavljen je u unutrašnjost glatkog polukružnog prstena, poluprečnika $R=L$, u vertikalnoj ravni. Na kom rastojanju $\overline{AE}$ treba dejstvovati silom $F=2G$, upravno na štap, da bi zauzeo položaj ravnoteže tako da se kraj A štapa nalazi na istoj vertikali sa centrom polukružnog prstena? Odrediti reakcije u tačkama oslanjanja štapa.



$\overline{AB}=2L$; G
$R=L$
$F=2G$

1) $\sum X_i=0$: $X_A=G\sin 45° \Rightarrow X_A=G\frac{\sqrt{2}}{2}$

$X_A=F_A\sin 45°=F_A\frac{\sqrt{2}}{2}$

$\Rightarrow F_A=G$

2) $\sum Y_i=0$: $Y_A-G\cos 45°-2G+F_D=0$

$\cancel{G\frac{\sqrt{2}}{2}}-\cancel{G\frac{\sqrt{2}}{2}}-2G+F_D=0 \Rightarrow F_D=2G$

3) $\sum M_A=0$: $G\cdot L\cos 45°+2G\cdot\overline{AE}-F_D\cdot L\sqrt{2}=0$

$\cancel{G}\cdot L\frac{\sqrt{2}}{2}+2\cancel{G}\cdot\overline{AE}-2\cancel{G}L\sqrt{2}=0$

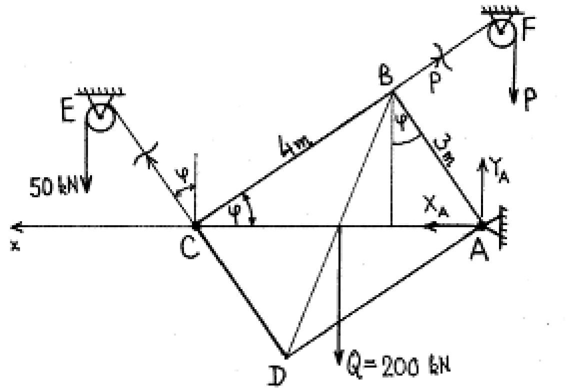
$\sqrt{2}L+4\overline{AE}-4\sqrt{2}L=0$

$4\overline{AE}=3\sqrt{2}L$

$\overline{AE}=\frac{3\sqrt{2}}{4}L\approx 1{,}056\,L$

ZADATAK BR. 15

Homogena pravougaona ploča ABCD, težine Q, može se obrtati u vertikalnoj ravni oko nepomičnog zgloba A. Ploča se održava u datom ravnotežnom položaju pomoću užadi koja su prebačena preko glatkih nepokretnih koturova E i F i zategnuta silama. Odrediti intenzitet sile P i komponente otpora zgloba A, kada je $Q = 200$ kN, levo uže zategnuto silom od 50 [kN] i $\cos\varphi = \frac{4}{5}$.



$\cos\varphi = \frac{4}{5} \Rightarrow \sin\varphi = \frac{3}{5}$

$\overline{AC} = d = 5\,m$

1) $\sum X_i = 0 : \; 50\cdot\sin\varphi - P\cos\varphi + X_A = 0$

2) $\sum Y_i = 0 : \; 50\cos\varphi + P\sin\varphi - Q + Y_A = 0$

3) $\sum \overset{\curvearrowleft}{M_A} = 0 : \; 50\cdot4 + P\cdot3 - 200\cdot2{,}5 = 0 \Rightarrow \underline{P = 100\,[kN]}$

1) $\Rightarrow X_A = -50\cdot\frac{3}{5} + 100\cdot\frac{4}{5}$; $\underline{X_A = 50\,[kN]}$

2) $\Rightarrow Y_A = 200 - 50\cdot\frac{4}{5} - 100\frac{3}{5}$; $\underline{Y_A = 100\,[kN]}$

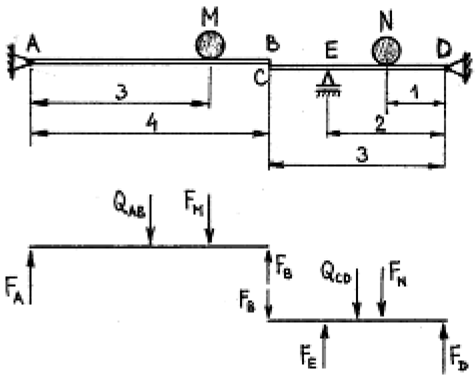
$F_A = \sqrt{X_A^2 + Y_A^2} = \sqrt{50^2 + 100^2} = 50\sqrt{1+2^2} = 50\sqrt{5}\;[kN]$

$tg\,\alpha = \frac{Y_A}{X_A} = \frac{100}{50} = 2$

$\underline{\alpha = arctg\,2 = 63^\circ 30'}$; $\underline{F_A \approx 112\,[kN]}$

ZADATAK BR. 16

Homogena prizmatična greda AB, dužine 4 m, težine 20 kN, vezana je u tački A zglavkasto za zid, a krajem B oslanja se na drugu homogenu prizmatičnu gredu CD, dužine 3 m, težine 16 kN. Greda CD je poduprta u tački E, a u tački D je zglavkasto vezana za zid. U tačkama M i N dejstvuju tereti, svaki po 8 kN. Odrediti otpore oslonaca.



$\overline{AB}$: 1) $\sum Y_i = 0$: $F_A - Q_{AB} - F_M + F_B = 0$

2) $\sum \overset{\curvearrowright}{M}_B = 0$: $F_A \cdot 4 - Q_{AB} \cdot 2 - F_M \cdot 1 = 0 \Rightarrow F_A = \frac{2Q_{AB} + F_M}{4} = \frac{2 \cdot 20 + 8}{4}$; $F_A = 12\ [kN]$

1) $\Rightarrow F_B = Q_{AB} + F_M - F_A = 20 + 8 - 12 \Rightarrow F_B = 16\ [kN]$

$\overline{BD}$: 3) $\sum Y_i = 0$: $-F_B + F_E - Q_{CD} - F_N + F_D = 0$

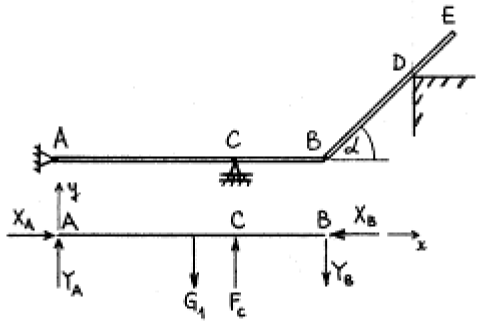
4) $\sum \overset{\curvearrowright}{M}_D = 0$: $-F_B \cdot 3 + F_E \cdot 2 - Q_{CD} \cdot 1{,}5 - F_N \cdot 1 = 0$

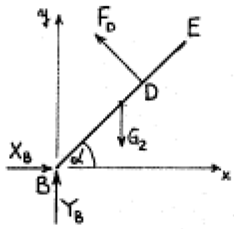
$$F_E = \frac{1}{2}\left(3F_B + Q_{CD} \cdot 1{,}5 + F_N\right) = \frac{1}{2}\left(3 \cdot 16 + 16 \cdot 1{,}5 + 8\right) \Rightarrow F_E = 40\ [kN]$$

3) $\Rightarrow F_D = F_B + Q_{CD} + F_N - F_E = 16 + 16 + 8 - 40 \Rightarrow F_D = 0$

ZADATAK BR. 17

Horizontalna greda $\overline{AB}$, težine $G_1 = 20$ kN, pričvršćena je za zid zglobom A i oslanja se na oslonac C. Za kraj B grede zglobom je pričvršćena greda $\overline{BE}$, težine $G_2 = 40$ kN, koja se oslanja na ispust D. Pri tome je: $\overline{CB} = \frac{1}{3}\overline{AB}$, $\overline{DE} = \frac{1}{3}\overline{BE}$ i $\alpha = 45°$. Odrediti reakcije oslonaca.





GREDA $\overline{BE}$:

1) $\sum X_i = 0: \quad X_B - F_D \sin\alpha = 0$

$X_B = F_D \sin\alpha$

2) $\sum Y_i = 0: \quad Y_B - G_2 + F_D \cos\alpha = 0$

$Y_B = G_2 - F_D \cos\alpha$

3) $\sum M_B = 0: \quad F_D \cdot \frac{2}{3}\overline{BE} - G_2 \frac{1}{2}\overline{BE}\cos\alpha = 0$

$F_D = \frac{3}{2} G_2 \frac{1}{2} \cos\alpha$

$F_D = \frac{3}{4} \cdot 40 \cdot \frac{\sqrt{2}}{2}$

$F_D = 21{,}2$ [kN]

1) $\Rightarrow X_B = 15$ [kN]

2) $\Rightarrow Y_B = 25$ [kN]

$F_B = \sqrt{X_B^2 + Y_B^2} = 29{,}15$ [kN]

GREDA $\overline{AB}$:

1) $\sum X_i = 0: \quad X_A - X_B = 0 \Rightarrow X_A = X_B = 15$ [kN]

2) $\sum Y_i = 0: \quad Y_A - G_1 + F_C - Y_B = 0$

3) $\sum M_A = 0: \quad -Y_B \cdot \cancel{\overline{AB}} + F_C \frac{2}{3}\cancel{\overline{AB}} - G_1 \frac{1}{2}\cancel{\overline{AB}} = 0$

$F_C = \frac{3}{2}\left(\frac{1}{2}G_1 + Y_B\right) = \frac{3}{2}\left(\frac{1}{2} \cdot 20 + 25\right)$

$F_C = 52{,}5$ [kN]

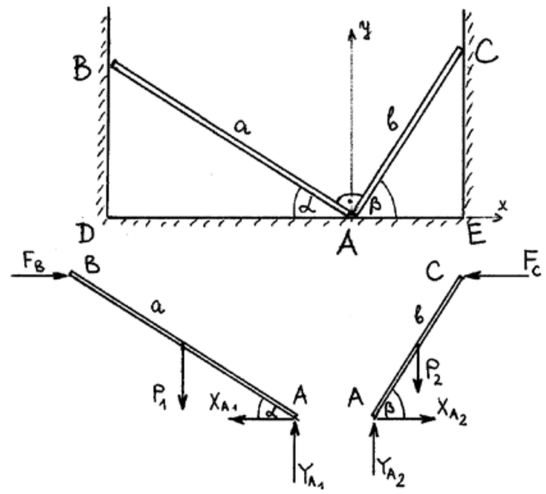
2) $\Rightarrow Y_A = G_1 - F_C + Y_B = 20 - 52{,}5 + 25$

$Y_A = -7{,}5$ [kN]

$F_A = \sqrt{X_A^2 + Y_A^2} = 16{,}77$ [kN]

ZADATAK BR. 18

Dva homogena štapa $\overline{AB}$ i $\overline{AC}$ oslanjaju se u tački A na glatki horizontalni zid i naslanjaju se jedan na drugi. U tački B, odnosno C, štapovi se oslanjaju na vertikalne zidove.
Odrediti razmak $\overline{DE}$ između zidova pri kojem ose štapova, u ravnotežnom položaju sistema, zaklapaju među sobom ugao od 90°.
Dužina štapa $\overline{AB} = a$, $\overline{AC} = b$. Težina štapa $\overline{AB}$ jednaka je P_1, a štapa $\overline{AC}$ jednaka je P_2.



$$\overline{DE} = a\cos\alpha + b\cos\beta$$

$$\alpha + \beta = 90° \Rightarrow \beta = 90° - \alpha,$$

$$\cos\beta = \cos(90-\alpha) = \sin\alpha,$$

$$\underline{\overline{DE} = a\cos\alpha + b\sin\alpha} \quad (*)$$

$$X_{A_1} = X_{A_2} \Rightarrow F_B = F_C \quad (**)$$

$$Y_A = Y_{A_1} + Y_{A_2} = P_1 + P_2$$

$\overline{AB}$:

$\Sigma M_B = 0 : P_1 \frac{a}{2}\cos\alpha - Y_{A_1} a\cos\alpha + X_{A_1} a\sin\alpha = 0;$

$\Sigma X_i = 0 : F_B - X_{A_1} = 0 \Rightarrow \underline{F_B = X_{A_1}}$

$\Sigma Y_i = 0 : Y_{A_1} - P_1 = 0 \Rightarrow \underline{Y_{A_1} = P_1}$

$\Sigma M_A = 0 : F_B a\sin\alpha - P_1 \frac{a}{2}\cos\alpha = 0,$

$$\underline{F_B = P_1 \frac{\cos\alpha}{2\sin\alpha} = X_{A_1}}$$

$\overline{AC}$:

$\Sigma M_C = 0 ; P_2 \frac{b}{2}\cos\beta - Y_{A_2} b\cos\beta + X_{A_2} b\sin\beta = 0$

$\Sigma X_i = 0 ; F_C - X_{A_2} = 0 \Rightarrow \underline{F_C = X_{A_2}}$

$\Sigma Y_i = 0 : Y_{A_2} - P_2 = 0 \Rightarrow \underline{Y_{A_2} = P_2}$

$\Sigma M_A = 0 : F_C \cdot b\sin\beta - P_2 \frac{b}{2}\cos\beta = 0,$

$$\underline{F_C = P_2 \frac{\cos\beta}{2\sin\beta} = X_{A_2}}$$

$$(**) \Rightarrow P_1 \frac{\cos\alpha}{\cancel{2}\sin\alpha} = P_2 \frac{\cos\beta}{\cancel{2}\sin\beta}$$

$$P_1 \operatorname{tg}\beta = P_2 \operatorname{tg}\alpha \; ; \; \alpha + \beta = 90° \Rightarrow P_1 \operatorname{ctg}\alpha = P_2 \operatorname{tg}\alpha,$$

$$\frac{1}{\operatorname{tg}^2\alpha} = \frac{P_2}{P_1} \Rightarrow \boxed{\operatorname{tg}\alpha = \sqrt{\frac{P_1}{P_2}}}$$

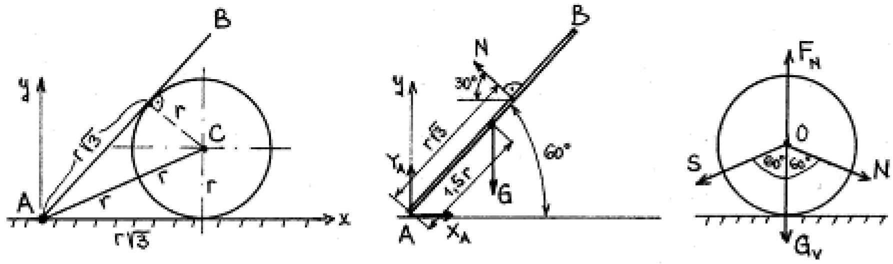
$$\cos\alpha = \frac{1}{\sqrt{1 + \operatorname{tg}^2\alpha}} = \frac{1}{\sqrt{1 + \frac{P_1}{P_2}}} = \frac{\sqrt{P_2}}{\sqrt{P_1 + P_2}}$$

$$\sin\alpha = \frac{\operatorname{tg}\alpha}{\sqrt{1 + \operatorname{tg}^2\alpha}} = \frac{\sqrt{\frac{P_1}{P_2}}}{\sqrt{1 + \frac{P_1}{P_2}}} = \frac{\sqrt{P_1}}{\sqrt{P_1 + P_2}}$$

$$(*) \Rightarrow \left\{ \overline{DE} = a\frac{\sqrt{P_2}}{\sqrt{P_1 + P_2}} + b\frac{\sqrt{P_1}}{\sqrt{P_1 + P_2}} = \frac{a\sqrt{P_2} + b_1\sqrt{P_1}}{\sqrt{P_1 + P_2}} \right\}$$

ZADATAK BR. 19

Homogeni prizmatični štap $\overline{AB}$, koji se može obrtati oko horizontalne ose A, oslanja se na omotač glatkog valjka, prečnika 2r. Valjak leži na horizontalnoj ravni, a vezan je pomoću kanapa $\overline{AC}$ za tačku A. Težina štapa iznosi 16 kN. Dužina štapa $\overline{AB} = 3r$, a kanapa $\overline{AC} = 2r$.
Odrediti silu S u koncu, pritisak štapa na kuglu i otpor zgloba A.



1) $\Sigma X_i = 0: \quad X_A - N\cos 30° = 0$

2) $\Sigma Y_i = 0: \quad Y_A - G + N\sin 30° = 0$

3) $\Sigma M_A = 0: \quad N \cdot r\sqrt{3} - G \cdot 1{,}5r\cos 60° = 0 \quad \Rightarrow \quad N = G\frac{1{,}5\cancel{r}}{\cancel{r}\sqrt{3}} \cdot \frac{1}{2} = \frac{3}{4}\frac{G}{\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}}$

$$N = G\frac{\sqrt{3}}{4} = 4\sqrt{3}\ [kN]$$

1) $\Rightarrow X_A = N\cos 30° = G\frac{\sqrt{3}}{4} \cdot \frac{\sqrt{3}}{2}$; $\quad X_A = \frac{3}{8}G = 6\ [kN]$

2) $\Rightarrow Y_A = G - N\sin 30° = G - G\frac{\sqrt{3}}{4} \cdot \frac{1}{2}$; $\quad Y_A = G\frac{(8-\sqrt{3})}{8} = 12{,}54\ [kN]$

$F_A = \sqrt{X_A^2 + Y_A^2} \quad \Rightarrow \quad F_A = 13{,}9\ [kN]$

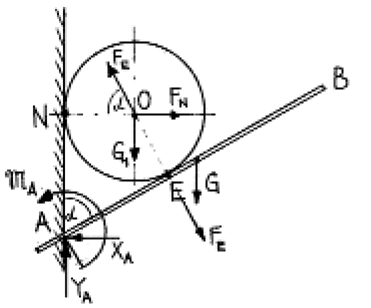
$tg\,\alpha = \frac{Y_A}{X_A} = \frac{12{,}54}{6} = 2{,}09 \quad \Rightarrow \quad \alpha = 64{,}4°$

$S\sin 60° = N\sin 60° \quad \Rightarrow \quad S = N = 4\sqrt{3}\ [kN]$

$F_N = G_v + 2N\cos 60° = G_v + N$

ZADATAK BR. 20

Greda $\overline{AB}$, dužine $l = 0,8\,m$ i težine $G = 100\,kN$, uzidana je u zid i to pod uglom od $\alpha = 60°$. Na njoj leži cilindar težine $G_1 = 180\,kN$, koji dodiruje gredu u tački E, pri čemu je $\overline{AE} = a = 0,3\,m$. Odrediti reakcije veza.



CILINDAR:

1) $\sum X_i = 0 : \quad F_N - F_E \cos\alpha = 0 \Rightarrow F_N = \frac{1}{2} F_E$

2) $\sum Y_i = 0 : \quad F_E \sin\alpha - G_1 = 0 \Rightarrow F_E = \frac{G_1}{\sin\alpha} = \frac{2\sqrt{3}}{3} G_1$

$F_E = 207,85\,[kN]$; $F_N = 103,92\,[kN]$

GREDA $\overline{AB}$:

1) $\sum X_i = 0 : \quad F_E \cos\alpha - X_A = 0 \Rightarrow X_A = \frac{G_1}{\sin\alpha} \cdot \cos\alpha = G_1 ctg\alpha$; $X_A = 103,92\,[kN]$

2) $\sum Y_i = 0 : \quad Y_A - F_E \sin\alpha - G = 0 \Rightarrow Y_A = G + F_E \sin\alpha = G + \frac{G_1}{\sin\alpha} \cdot \sin\alpha$,

$Y_A = G + G_1$; $Y_A = 280\,[kN]$

$F_A = \sqrt{X_A^2 + Y_A^2} = 298,67\,[kN]$

3) $\sum M_A = 0 : \quad \mathfrak{M}_A - F_E \cdot a - G \frac{l}{2} \sin\alpha = 0$

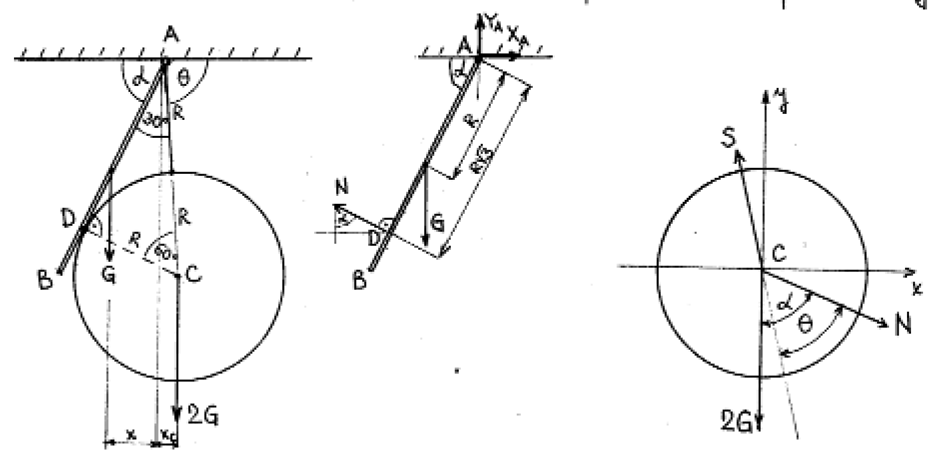
$\mathfrak{M}_A = G_1 \frac{a}{\sin\alpha} + G \frac{l}{2} \sin\alpha$

$\mathfrak{M}_A \approx 97\,[kNm]$

ZADATAK BR. 21

Homogeni štap $\overline{AB}$, težine G, dužine $2R$, zglobno je vezan u tački A, a u tački D oslanja se o glatku kuglu, poluprečnika R i težine $2G$. Kugla je vezana nerastegljivim užetom dužine R za tačku A.

Odrediti ugao α koji gradi štap sa horizontalnom osom u ravnotežnom položaju, reakciju zgloba A i pritisak štapa na kuglu.



1) $\sum X_i = 0:\ X_A - N\sin\alpha = 0$

2) $\sum Y_i = 0:\ Y_A - G + N\cos\alpha = 0$

3) $\sum M_A = 0:\ N\cdot \cancel{R}\sqrt{3} - G\cancel{R}\cos\alpha = 0 \ \Rightarrow\ N = \frac{G\sqrt{3}}{3}\cos\alpha$

1) $\Rightarrow\ X_A = N\sin\alpha \ \Rightarrow\ X_A = \frac{G\sqrt{3}}{3}\sin\alpha\cos\alpha$

2) $\Rightarrow\ Y_A = G - N\cos\alpha = G - \frac{G\sqrt{3}}{3}\cos^2\alpha = G\left(1 - \frac{\sqrt{3}}{3}\cos^2\alpha\right)$

1) $\sum X_i = 0:\ N\sin\alpha - S\cos\theta = 0$

2) $\sum Y_i = 0:\ S\sin\theta - N\cos\alpha - 2G = 0$

1) $\Rightarrow\ S = N\frac{\sin\alpha}{\cos\theta}$;

$\theta = 180^\circ - (\alpha + 30^\circ)$

$\sum M_A = 0:\ 2\cancel{G}x_c - \cancel{G}x = 0 \ \Rightarrow\ 2x_c = x$

$x = R\cos\alpha \ \Rightarrow\ x_c = \frac{1}{2}x = \frac{1}{2}R\cos\alpha \quad (*)$

$x_c = 2R\cos\theta = 2R\cos\left[180^\circ - (\alpha + 30^\circ)\right] = -2R\cos(\alpha + 30^\circ) =$

$= -2R(\cos\alpha\cos 30^\circ - \sin\alpha\sin 30^\circ) = -R(\sqrt{3}\cos\alpha - \sin\alpha)$

$x_c = R(\sin\alpha - \sqrt{3}\cos\alpha)$

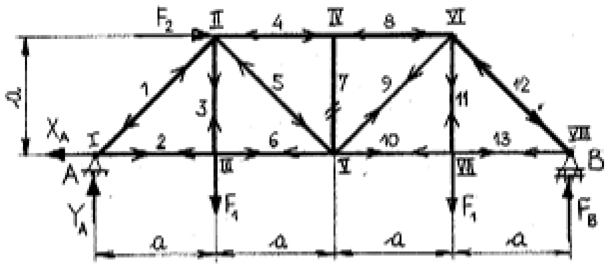
$(*) \Rightarrow \frac{1}{2}\cancel{R}\cos\alpha = \cancel{R}(\sin\alpha - \sqrt{3}\cos\alpha)$

$\sin\alpha = \left(\frac{1}{2} + \sqrt{3}\right)\cos\alpha \ /:\cos\alpha$

$\mathrm{tg}\,\alpha = \frac{1}{2} + \sqrt{3} \ \Rightarrow\ \alpha = \mathrm{arctg}\left(\frac{1}{2} + \sqrt{3}\right) \ \Rightarrow\ \alpha = 65^\circ 5' = 65{,}84^\circ$

ZADATAK BR. 22

Za ravanski rešetkasti nosač prikazan na slici analitički odrediti otpore oslonaca i sile u štapovima.



$F_1 = 40\ [kN]$, $F_2 = 60\ [kN]$, $a = 2\ [m]$

a) Analitički metod određivanja otpora oslonaca

1) $\sum X_i = 0: \quad -X_A + F_2 = 0 \Rightarrow X_A = F_2 = 60\ [kN]$

2) $\sum Y_i = 0: \quad Y_A - 2F_1 + F_B = 0$

3) $\sum M_B = 0: \quad Y_A \cdot 4a - F_1 \cdot 3a - F_1 \cdot a + F_2 \cdot a = 0 \quad /:a$

$$Y_A = F_1 - \frac{F_2}{4} = 40 - \frac{60}{4} \Rightarrow Y_A = 25\ [kN]$$

$$F_A = \sqrt{X_A^2 + Y_A^2} \Rightarrow F_A = 65\ [kN]$$

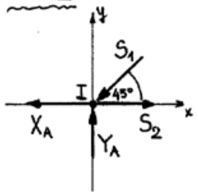
$$\alpha = \mathrm{arctg}\frac{Y_A}{-X_A} = \mathrm{arctg}\left(-\frac{5}{12}\right)$$

2) $\Rightarrow F_B = 2F_1 - Y_A; \quad F_B = 55\ [kN]$

* Određivanje unutrašnjih sila (sila u štapovima rešetke);

- analitička metoda ravnoteže čvorova -

čvor I



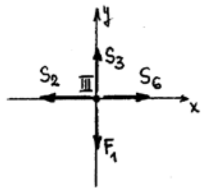
1) $\Sigma X_i = 0 \; : \; -X_A - S_1\cos 45^\circ + S_2 = 0$

2) $\Sigma Y_i = 0 \; : \; Y_A - S_1 \sin 45^\circ = 0$

$$S_1 = \frac{Y_A}{\sin 45^\circ} \Rightarrow S_1 = 25\sqrt{2}\ [kN]$$

$$S_2 = X_A + S_1\cos 45^\circ = 60 + 25\sqrt{2}\,\frac{\sqrt{2}}{2} \Rightarrow S_2 = 85\ [kN]$$

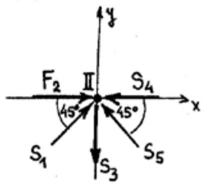
čvor III



1) $\Sigma X_i = 0 \; : \; -S_2 + S_6 = 0 \Rightarrow S_6 = S_2 = 85\ [kN]$

2) $\Sigma Y_i = 0 \; : \; S_3 - F_1 = 0 \Rightarrow S_3 = F_1 = 40\ [kN]$

čvor II



1) $\Sigma X_i = 0 : \; F_2 + S_1\cos 45^\circ - S_4 - S_5\cos 45^\circ = 0$

$$F_2 + S_1\frac{\sqrt{2}}{2} - S_4 - S_5\frac{\sqrt{2}}{2} = 0$$

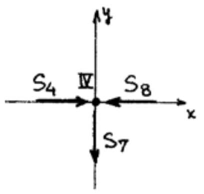
$$S_4 + S_5\frac{\sqrt{2}}{2} = 85$$

2) $\Sigma Y_i = 0 : \; S_1 \sin 45^\circ + S_5 \sin 45^\circ - S_3 = 0$

$$S_5 = \frac{S_3}{\sin 45^\circ} - S_1 = 40\sqrt{2} - 25\sqrt{2} \Rightarrow S_5 = 15\sqrt{2}\ [kN]$$

$$1) \Rightarrow S_4 = 85 - 15\cancel{\sqrt{2}}\,\frac{\cancel{\sqrt{2}}}{\cancel{2}} \Rightarrow S_4 = 70\ [kN]$$

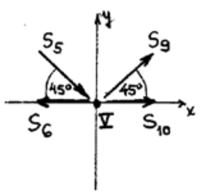
čvor IV



1) $\Sigma X_i = 0 : \; S_4 - S_8 = 0 \Rightarrow S_8 = S_4 = 70\ [kN]$

2) $\Sigma Y_i = 0 : \; -S_7 = 0 \Rightarrow S_7 = 0$

čvor V



1) $\Sigma X_i = 0 \; : \; -S_6 + S_5\cos 45^\circ + S_9\cos 45^\circ + S_{10} = 0$

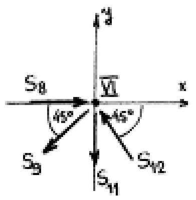
$$-85 + 15 + S_9\frac{\sqrt{2}}{2} + S_{10} = 0$$

$$S_9\frac{\sqrt{2}}{2} + S_{10} = 70$$

2) $\Sigma Y_i = 0 \quad -S_5 \cancel{\sin 45^\circ} + S_9 \cancel{\sin 45^\circ} = 0 \Rightarrow S_9 = S_5 = 15\sqrt{2}\ [kN]$

$$1) \Rightarrow S_{10} = 70 - 15\sqrt{2}\,\frac{\sqrt{2}}{2} \Rightarrow S_{10} = 55\ [kN]$$

čvor VI



1) $\sum X_i = 0$: $S_8 - S_9 \cos 45° - S_{12} \cos 45° = 0$

$$S_{12}\frac{\sqrt{2}}{2} = S_8 - S_9\frac{\sqrt{2}}{2}$$

$$S_{12} = S_8\sqrt{2} - S_9 = 70\sqrt{2} - 15\sqrt{2}$$

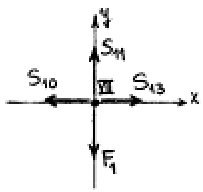
$$S_{12} = 55\sqrt{2}\ [kN]$$

2) $\sum Y_i = 0$: $-S_9 \sin 45° - S_{11} + S_{12} \sin 45° = 0$

$$S_{11} = S_{12}\frac{\sqrt{2}}{2} - S_9\frac{\sqrt{2}}{2} = 55\sqrt{2}\frac{\sqrt{2}}{2} - 15\sqrt{2}\frac{\sqrt{2}}{2}$$

$$S_{11} = 40\ [kN]$$

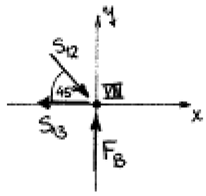
čvor VII



1) $\sum X_i = 0$: $-S_{10} + S_{13} = 0 \Rightarrow S_{13} = S_{10} = 55\ [kN]$

2) $\sum Y_i = 0$: $S_{11} - F_1 = 0 \Rightarrow S_{11} = F_1 = 40\ [kN]$

čvor VIII - PROVERA



1) $\sum X_i = 0$: $S_{12} \cos 45° - S_{13} = 0$

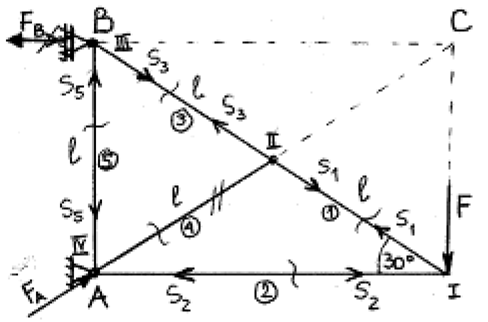
$$55\cancel{\sqrt{2}}\frac{\cancel{\sqrt{2}}}{2} - 55 = 0 \ ✓$$

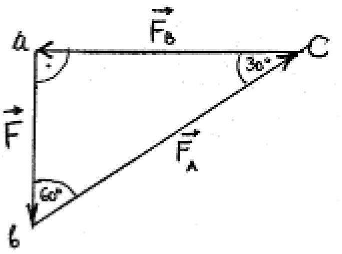
2) $\sum Y_i = 0$: $F_B - S_{12} \sin 45° = 0$

$$55 - 55\cancel{\sqrt{2}}\frac{\cancel{\sqrt{2}}}{2} = 0 \ ✓$$

ZADATAK BR. 23

Odrediti sile u štapovima date rešetke opterećene silom F kao na slici.





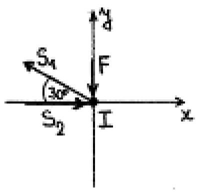
$$\frac{F}{\sin 30^\circ} = \frac{F_A}{\sin 90^\circ} = \frac{F_B}{\sin 60^\circ}$$

s – br. štapova
n – br. čvorova

$$s = 2n - 3$$
$$s = 2 \cdot 4 - 3 \Rightarrow s = 5$$

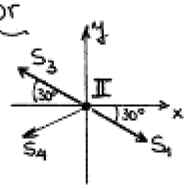
$$F_A = F\,\frac{\sin 90^\circ}{\sin 30^\circ} \Rightarrow F_A = 2F$$
$$F_B = F\,\frac{\sin 60^\circ}{\sin 30^\circ} \Rightarrow F_B = \sqrt{3}\,F$$

I čvor



1) $\Sigma X_i = -S_1\cos 30^\circ + S_2 = 0$
$$S_2 = S_1\frac{\sqrt{3}}{2}$$
2) $\Sigma Y_i = S_1\sin 30^\circ - F = 0$
$$S_1 = 2F\;;\quad S_2 = F\sqrt{3}$$

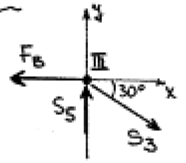
II čvor



1) $\Sigma X_i = S_1\cancel{\cos 30^\circ} - S_3\cancel{\cos 30^\circ} - S_4\cancel{\cos 30^\circ} = 0$
2) $\Sigma Y_i = -S_1\cancel{\sin 30^\circ} + S_3\cancel{\sin 30^\circ} - S_4\cancel{\sin 30^\circ} = 0$ } +

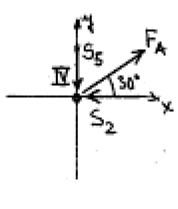
$$2S_4 = 0 \Rightarrow S_4 = 0\;;\quad S_3 = S_1 = 2F$$

III čvor



1) $\Sigma X_i = S_3\cos 30^\circ - F_B = 0$
$$F_B = S_3\frac{\sqrt{3}}{2} = 2F\frac{\sqrt{3}}{2} \Rightarrow F_B = F\sqrt{3}$$
2) $\Sigma Y_i = S_5 - S_3\sin 30^\circ = 0$
$$S_5 = S_3\cdot\frac{1}{2} \Rightarrow S_5 = F$$

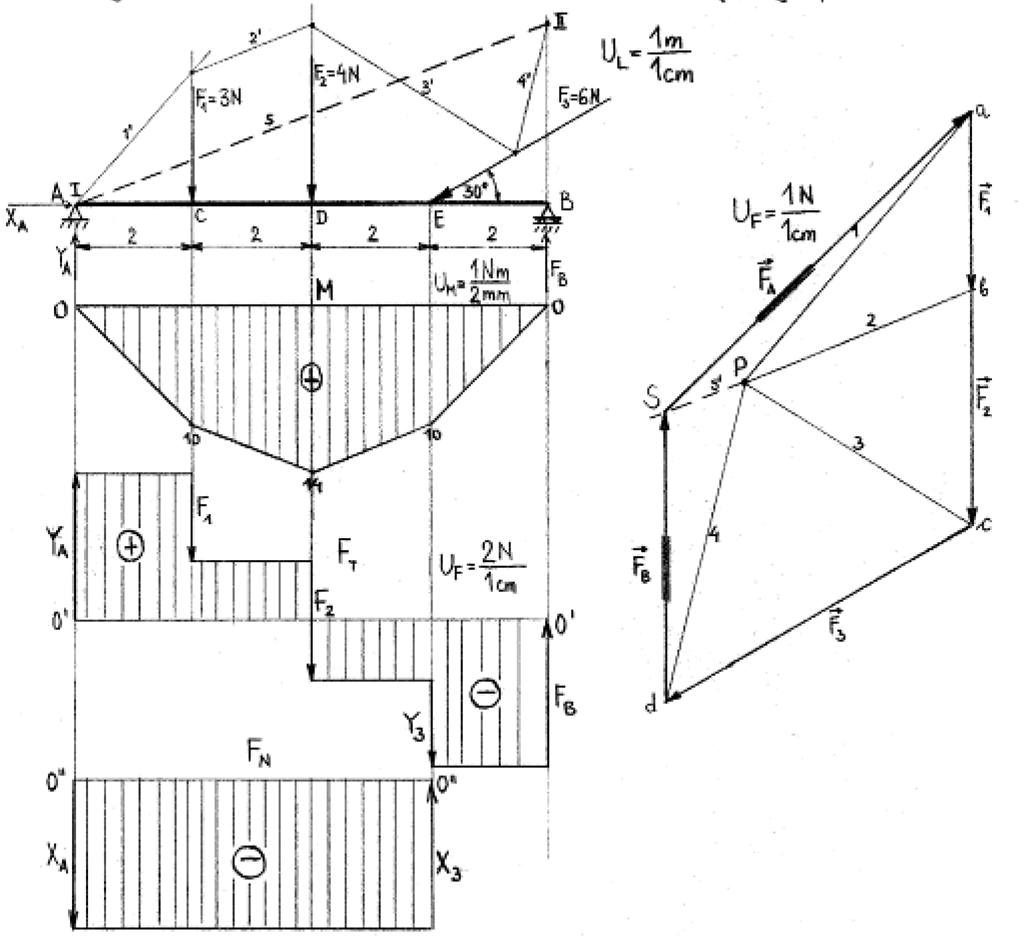
IV čvor – PROVERA:



1) $\Sigma X_i = F_A\cos 30^\circ - S_2 = 0$
$$\cancel{2}F\frac{\sqrt{3}}{\cancel{2}} - F\sqrt{3} = 0 \Rightarrow 0 = 0 \;✓$$
2) $\Sigma Y_i = F_A\sin 30^\circ - S_5 = 0$
$$\frac{1}{\cancel{2}}\cancel{2}F - F = 0 \Rightarrow 0 = 0 \;✓$$

ZADATAK BR. 24

Prosta greda, raspona 8 [m], opterećena je datim teretima. Računski i grafički odrediti otpore oslonaca i nacrtati dijagrame napadnog momenta, transverzalne i aksijalne sile. Koliki je najveći napadni moment i u kom preseku dejstvuje?



1) $\Sigma X_i = 0$; $X_A - F_3 \cos 30° = 0 \Rightarrow X_A = F_3 \cos 30° = 6\frac{\sqrt{3}}{2} \Rightarrow X_A = 5{,}2\ [N]$

2) $\Sigma Y_i = 0$; $Y_A - F_1 - F_2 - F_3 \sin 30° + F_B = 0 \Rightarrow Y_A + F_B = 10$

3) $\Sigma M_B = 0$; $Y_A \cdot 8 - F_1 \cdot 6 - F_2 \cdot 4 - F_3 \sin 30° \cdot 2 = 0$

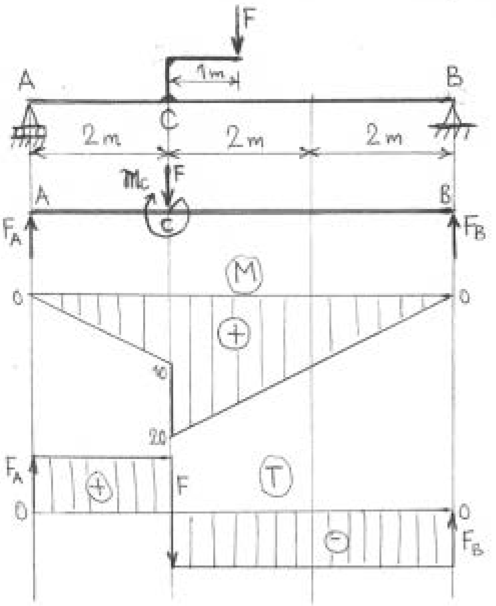
$8Y_A = 6F_1 + 4F_2 + F_3 \Rightarrow Y_A = 5\ [N]$; $\boxed{F_B = 5\ [N]}$

$F_A = \sqrt{X_A^2 + Y_A^2} \Rightarrow \boxed{F_A = 7{,}2\ [N]}$; $tg\theta = \frac{Y_A}{X_A} \Rightarrow \theta = 43{,}88°$

$M_C^l = Y_A \cdot 2 = 10\ [Nm]$; $M_D^l = Y_A \cdot 4 - F_1 \cdot 2 = 14\ [Nm] = M_{max}$

ZADATAK BR. 25

Odrediti otpore oslonaca i nacrtati statičke dijagrame proste grede koja je u tački C ekscentrično opterećena vertikalnom silom intenziteta $F = 10\ [N]$.



$\mathfrak{M}_C = F \cdot 1 = 10\ Nm$

1) $\sum X_i = 0$

2) $\sum Y_i = 0: \ F_A + F_B = 10$

3) $\sum M_A^{F_i} = 0: \ F_B \cdot 6 - F \cdot 2 - \mathfrak{M}_c = 0$

$6F_B = 2F + \mathfrak{M}_c$

$6F_B = 30$

$\underline{F_B = 5\ N}$; $\underline{F_A = 5\ N}$

$M_A = 0$

$M_{C,l}^{l} = F_A \cdot 2 = 10\ Nm$

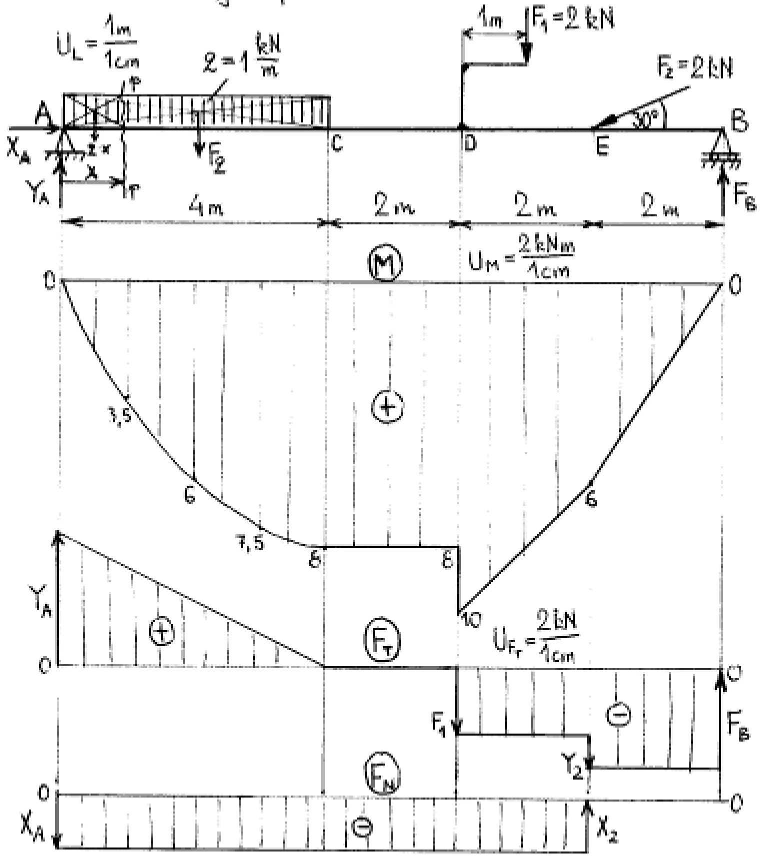
$M_{C,d}^{l} = F_A \cdot 2 + \mathfrak{M}_C = 20\ Nm$

$M_{C,d}^{d} = F_B \cdot 4 = 20\ Nm$

$M_B = 0$

ZADATAK BR. 26

Nosač AB je opterećen datim teretima. Nacrtati statičke dijagrame.



1) $\sum X_i = 0: \quad X_A - F_2 \cos 30° = 0 \Rightarrow X_A = 2 \cdot \frac{\sqrt{3}}{2}$; $X_A = \sqrt{3}$ [kN]

2) $\sum Y_i = 0: \quad Y_A - F_q - F_1 - F_2 \sin 30° + F_B = 0 \Rightarrow Y_A + F_B = q \cdot 4 + 2 + 2 \cdot \frac{1}{2}$; $F_q = q \cdot 4 = 4$ kN

$Y_A + F_B = 7$

3) $\sum M_B = 0: \quad Y_A \cdot 10 - F_q \cdot 8 - F_1 \cdot 3 - F_2 \sin 30° \cdot 2 = 0$

$10\, Y_A = 32 + 6 + 2 = 40 \Rightarrow$ $Y_A = 4$ [kN] , $F_B = 3$ [kN]

$F_A = \sqrt{X_A^2 + Y_A^2} = \sqrt{19} = 4{,}36$ [kN]

$\theta_{F_A} = \text{arctg}\, \frac{Y_A}{X_A} = 66{,}6°$

* Napadni momenti u karakterističnim tačkama grede :

$M_A = 0$

$M_C = Y_A \cdot 4 - F_2 \cdot 2 = 4 \cdot 4 - 4 \cdot 2 = 16 - 8 = 8\ [kNm]$

$M_D^l = Y_A \cdot 6 - F_2 \cdot 4 = 4 \cdot 6 - 4 \cdot 4 = 8\ [kNm]$

$M_D^d = Y_A \cdot 6 - F_2 \cdot 4 + \mathfrak{M}_D = 8 + F_1 \cdot 1 = 8 + 2 = 10\ [kNm]$, $\mathfrak{M}_D = F_1 \cdot 1 = 2\ [kNm]$

$M_E = F_B \cdot 2 = 3 \cdot 2 = 6\ [kNm]$

$M_B = 0$

* Transverzalne sile u poljima :

$A \div C$: $F_T = Y_A - 2 \cdot x \Big|_0^4$; $F_{TA}\Big|_{x=0} = Y_A = 4\ [kN]$; $F_{T_C}\Big|_{x=4m} = 0$

$D \div E$: $F_T = F_1 = -2\ [kN]$

$B \div E$: $F_T = F_B = 3\ [kN]$

* Napadni moment i transverzalna sila u preseku p-p :

$\overset{\curvearrowright}{M}_{p\text{-}p}(x) = Y_A \cdot x - 2 \cdot x \cdot \frac{1}{2} x = 4x - \frac{1}{2} x^2$; $x \in [0, 4]\ m$

$M(x = 1m) = 3{,}5\ [kNm]$

$M(x = 2m) = 6\ [kNm]$

$M(x = 3m) = 7{,}5\ [kNm]$

$M(x = 4m) = 8\ [kNm] = M_C$

$F_{T\,p\text{-}p}(x) = \dfrac{dM_{p\text{-}p}(x)}{dx} = 4 - x$

$F_T(x = 0) = Y_A = 4\ [kN]$

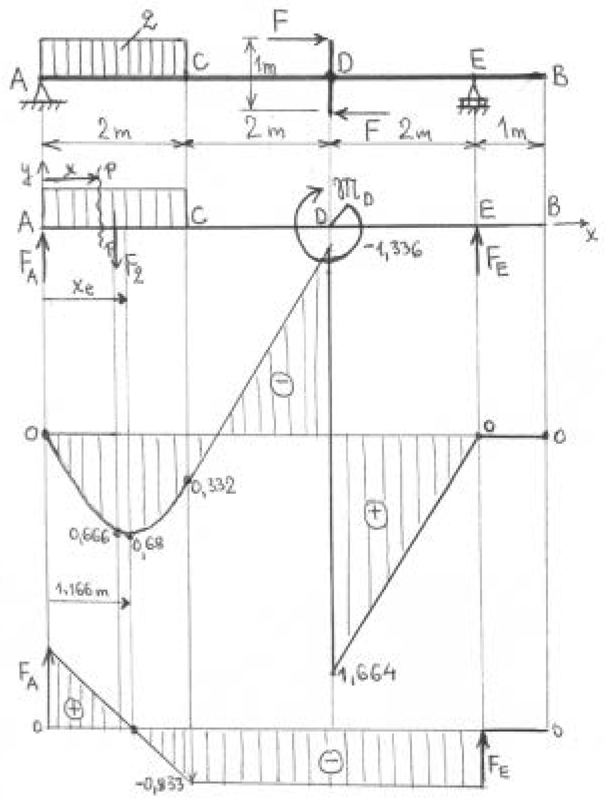
$F_T(x = 4m) = F_{T_C} = 0$

$\dfrac{dM(x)}{dx} = 0 \Rightarrow 4 - x_e = 0 \Rightarrow x_e = 4\ m$; $M(x_e) = M_{max}(x) = 8\ [kNm]$

* $M_{max} = M_D = 10\ [kNm]$

ZADATAK BR. 27

Odrediti otpore oslonaca i nacrtati statičke dijagrame proste grede $\overline{AB}$ koja je opterećena jednoliko raspodeljenim opterećenjem i spregom sila kao što je prikazano na slici. $q = 1$ [kN/m], F = 3 kN.



$\mathfrak{M}_D = F \cdot 1 = 3$ kNm

$F_q = q \cdot 2 = 2$ kN

1) $\sum X = 0$

2) $\sum Y = 0: \; F_A - F_q + F_E = 0$

$F_A + F_E = 2$

3) $\sum M_A = 0: \; F_E \cdot 6 - \mathfrak{M}_D - F_q \cdot 1 = 0$

$6F_E = \mathfrak{M}_D + F_q$

$6F_E = 5$

$F_E = \frac{5}{6} = 0{,}833$ kN

2) $\Rightarrow F_A = 1{,}166$ kN

$M_A = 0$

$M_C = F_A \cdot 2 - F_q \cdot 1 = 0{,}332$ kNm

$M^{\ell}_{D,\ell} = F_A \cdot 4 - F_q \cdot 3 = -1{,}336$ kNm

$M^{\ell}_{D,d} = F_A \cdot 4 - F_q \cdot 3 + \mathfrak{M}_D = 1{,}664$ kNm

$M^{d}_{D,d} = F_E \cdot 2 = 1{,}664$ kNm

$M_E = 0$

$M(x) = F_A \cdot x - q \cdot x \cdot \frac{x}{2} \;\; ; \; 0 \le x \le 2$ m

$M(x) = F_A x - \frac{1}{2} q x^2 = 1{,}166x - 0{,}5x^2$

$M(x=0) = M_A = 0$

$M(x=1) = 0{,}666$ kNm

$M(x=2) = M_C = 0{,}332$ kNm

$\frac{dM(x)}{dx} = 1{,}166 - x$

$\frac{dM(x)}{dx} = 0 \Rightarrow 1{,}166 - x_e = 0 \Rightarrow x_e = 1{,}166$ m

$x_e \in [0,2]$ m

$M(x_e) = 1{,}166 \cdot 1{,}166 - 0{,}5(1{,}166)^2$

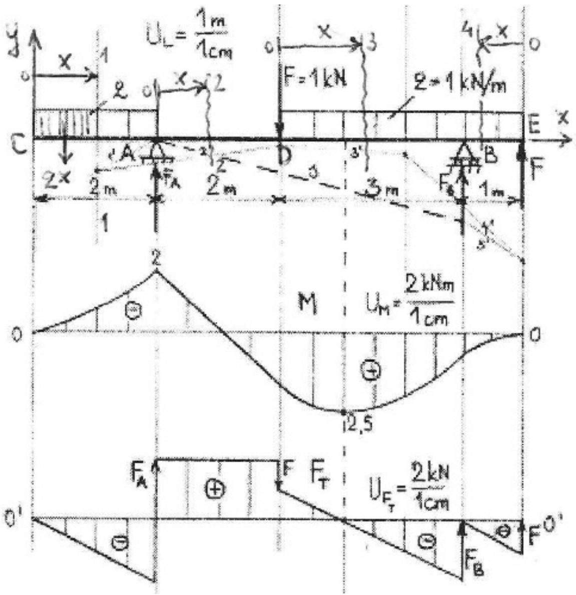
$M(x_e) = 0{,}68$ kNm

$M_{max} = M_{D,d} = 1{,}664$ kNm

$F_T(x) = \frac{dM(x)}{dx} = 1{,}166 - x \;\; ; \; 0 \le x \le 2$ m

ZADATAK BR. 28

Za prikazanu gredu sa prepustima odrediti otpore oslonaca, naći opasan presek i nacrtati statičke dijagrame.



1) $\sum Y_i = 0$: $-2\cdot 2 + F_A \cancel{-F} - 2\cdot 4 + F_B \cancel{+F} = 0 \Rightarrow F_A + F_B = 6$

2) $\sum M_A = 0$: $F\cdot 6 + F_B\cdot 5 - 42\cdot 4 - F\cdot 2 + 22\cdot 1 = 0 \Rightarrow 5F_B = 10 \Rightarrow F_B = 2\ kN$, $F_A = 4\ kN$;

POLJE $\overline{CA}$: $0 \le x \le 2\ m$

$M(x) = -2x\cdot\frac{x}{2} = -\frac{1}{2}x^2$; $T(x) = \frac{dM(x)}{dx} = -2x = -x$;

$M(0) = M_C = 0$ $\quad T(0) = 0$

$M(1) = -0{,}5\ kNm$ $\quad T(2) = 2\ kN$

$M(2) = -2\ kNm = M_A$

POLJE $\overline{AD}$: $0 \le x \le 2\ m$

$M(x) = -2\cdot 2(1+x) + F_A\cdot x = -2 - 2x + 4x = 2x - 2$; $T(x) = \frac{dM(x)}{dx} = 2\ kN = const.$

$M(0) = M_A = -2\ kNm$

$M(2) = M_D = 2\ kNm$

POLJE $\overline{DB}$: $0 \le x \le 3\ m$

$M(x) = -2\cdot 2(3+x) + F_A(2+x) - F\cdot x - 2\cdot x\cdot\frac{x}{2}$; $T(x) = \frac{dM(x)}{dx} = -x + 1$

$M(x) = -\frac{1}{2}x^2 + x + 2$

$M(0) = M_D = 2\ kNm$ $\quad T(0) = 1\ kN$

$M(1) = 2{,}5\ kNm$ $\quad T(3) = -2\ kN$.

$M(2) = 2\ kNm$

$M(3) = M_B = 0{,}5\ kNm$

$\frac{dM(x)}{dx} = 0 \Rightarrow -x + 1 = 0 \Rightarrow x_e = 1\ m$

$M(x_e) = M_{fmax} = 2{,}5\ kNm$

POLJE $\overline{EB}$: $0 \le x \le 1\ m$

$M(x) = F\cdot x - 2\cdot x\cdot\frac{x}{2} = x - \frac{1}{2}x^2$; $T(x) = -\frac{dM(x)}{dx} = -1 + x$

$M(0) = M_E = 0$ $\quad T(0) = -1\ kN$

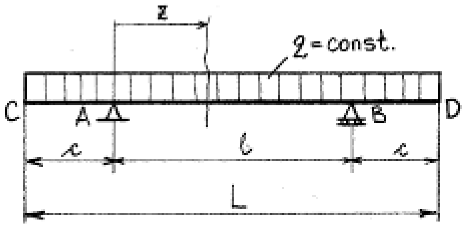
$M(1) = M_B = 0{,}5\ kNm$ $\quad T(1) = 0$.

ZADATAK BR. 29

Greda ABCD, dužine L, sa dva jednaka prepusta c, opterećena je po celoj dužini jednolikim opterećenjem $q = const.$

a) Na kom rastojanju l treba postaviti oslonce A i B da bi apsolutne vrednosti maksimalnih pozitivnih i negativnih napadnih momenata (tzv. granične vrednosti) bile jednake ?

b) U kom odnosu, u tom slučaju, stoje dužine prepusta c sa rasponom oslonca l ?



$L = l + 2c$

$F_A = F_B$

$F_q = qL$

a) $\sum Y_i = 0$: $F_A + F_B - qL = 0 \rightarrow \underline{F_A = F_B = \frac{1}{2} qL}$

$M_{min} = M_A = M_B = -\frac{1}{2} q c^2$; $c = \frac{1}{2}(L-l)$, $c^2 = \frac{1}{4}(L-l)^2$

$M_{min} = -\frac{1}{8} q (L-l)^2$

$$
\begin{aligned}
M_{max} = M\left(\tfrac{1}{2} l\right) &= -qc\tfrac{1}{2}(c+l) + \tfrac{1}{2}F_A \cdot l - q\tfrac{l}{2}\cdot\tfrac{l}{4} = \\
&= -\tfrac{1}{2}qc^2 - \tfrac{1}{2}qcl + \tfrac{1}{2}\cdot\tfrac{1}{2}qLl - \tfrac{1}{8}ql^2 = \\
&= -\tfrac{1}{2}q\tfrac{1}{4}(L-l)^2 - \tfrac{1}{2}q\tfrac{1}{2}(L-l)l + \tfrac{1}{4}qLl - \tfrac{1}{8}ql^2 = \\
&= -\tfrac{1}{8}q(L^2 - 2Ll + l^2) - \tfrac{1}{4}q(Ll - l^2) + \tfrac{1}{4}qLl - \tfrac{1}{8}ql^2 = \\
&= \tfrac{1}{8}qL(2l - L)
\end{aligned}
$$

$$|M_{min}| = |M_{max}| \Rightarrow \cancel{\tfrac{1}{8}q}(L-l)^2 = \cancel{\tfrac{1}{8}q}L(2l-L)$$

$$l^2 - 4Ll + 2L^2 = 0$$

$$l_{1/2} = \frac{4L \pm \sqrt{16L^2 - 8L^2}}{2} = 2L \pm L\sqrt{2}$$

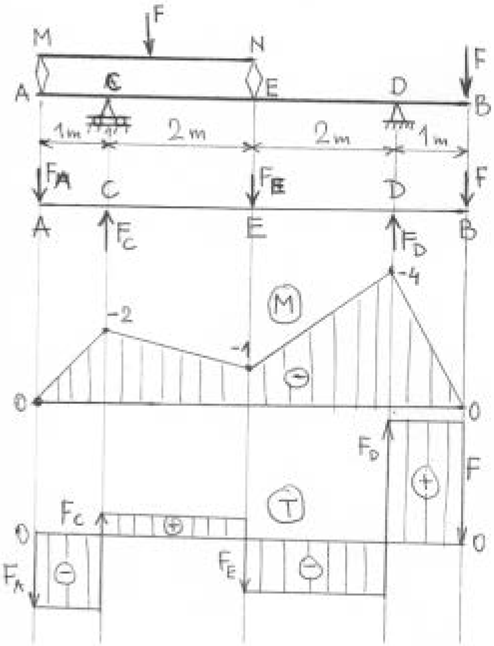
$$l < L \Rightarrow \underline{l = (2-\sqrt{2})L}$$

b) $L = l + 2c \Rightarrow l = (2-\sqrt{2})(l+2c)$

$$\frac{c}{l} = 0{,}347$$

ZADATAK BR. 30

Odrediti otpore oslonaca C i D proste grede $\overline{AB}$ posredno opterećene u tačkama A i E prostom gredom $\overline{MN}$ na koju na sredini raspona dejstvuje sila F, kao i silom F u tački B. F = 4 kN.



$\overline{MN}$: $F_M + F_N = F = 4$; $F_M = F_N = 2$ kN

$F_A = F_M = 2$ kN; $F_E = F_N = 2$ kN

$\overline{AB}$: 1) $\sum X_i = 0$

2) $\sum Y_i = 0$: $F_C + F_D = 8$

3) $\sum M_C^{F_i} = 0$: $-F \cdot 5 + F_D \cdot 4 - F_E \cdot 2 + F_A \cdot 1 = 0$

$$4F_D = 5F + 2F_E - F_A$$

$$4F_D = 20 + 4 - 2$$

$$4F_D = 22$$

$F_D = 5,5$ kN ; $F_C = 2,5$ kN

$M_A = 0$

$M_C = -F_A \cdot 1 = -2$ kNm

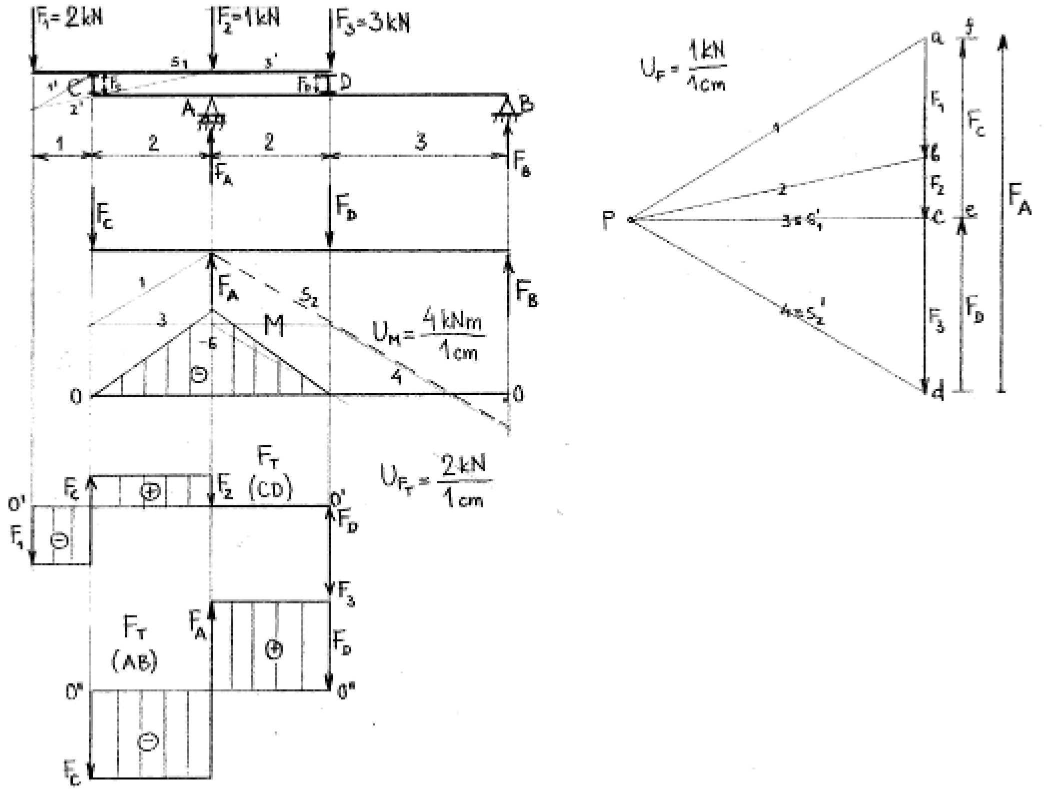
$M_E = -F_A \cdot 3 + F_C \cdot 2 = -6 + 5 = -1$ kNm

$M_D = -F \cdot 1 = -4$ kNm

$M_B = 0$

ZADATAK BR. 31

Za date opterećene grede, $F_1 = 2$ kN, $F_2 = 1$ kN, $F_3 = 3$ kN, računski i grafički odrediti otpore oslonaca. Dijagram napadnog momenta nacrtati za obe grede ujedno.



Greda CD: $\Sigma Y = 0$, $F_C + F_D - F_1 - F_2 - F_3 = 0 \Rightarrow F_C + F_D = 6$

$\Sigma M_D = 0$, $-F_1 \cdot 5 + F_C \cdot 4 - F_2 \cdot 2 = 0 \Rightarrow \underline{F_C = 3 \text{ kN}}$, $\underline{F_D = 3 \text{ kN}}$

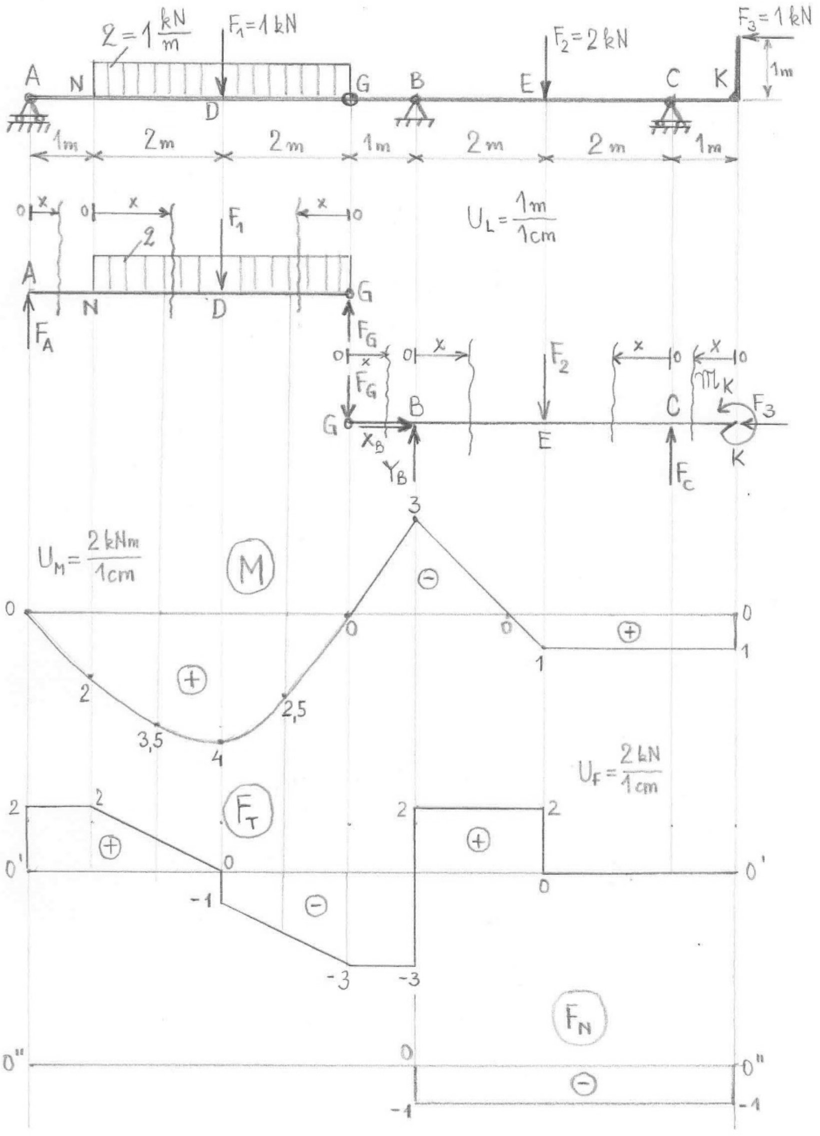
Greda AB: $\Sigma Y = 0$, $F_A + F_B = F_C + F_D \Rightarrow F_A + F_B = 6$

$\Sigma M_B = 0$, $-F_C \cdot 7 + F_A \cdot 5 - F_D \cdot 3 = 0 \Rightarrow \underline{F_A = 6 \text{ kN}}$, $\underline{F_B = 0}$

$M_A^l = -F_C \cdot 2 = -6$ kNm

ZADATAK BR. 32

Za dati Gerberov nosač odrediti otpore oslonaca i nacrtati statičke dijagrame.



GREDA $\overline{AG}$:

1) $\sum Y_i = 0$: $F_A + F_B - F_1 - F_2 = 0 \Rightarrow F_A + F_G = 5$

2) $\sum M_A = 0$: $5F_G - 3(F_2 + F_1) = 0 \Rightarrow F_G = \frac{3}{5}(F_2 + F_1)$; $F_G = 3$ kN , $F_A = 2$ kN ;

3) $M_G^l = M_G^d = 0$;

GREDA $\overline{BC}$:

1) $\sum X_i = 0$: $X_B - F_3 = 0 \Rightarrow X_B = 1$ kN ;

2) $\sum Y_i = 0$: $Y_B + F_C - F_G - F_2 = 0 \Rightarrow Y_B + F_C = 5$;

3) $\sum M_C = 0$: $-5F_G + 4Y_B - 2F_2 - F_3 = 0 \Rightarrow Y_B = 5$ kN , $F_C = 0$;

4) $M_G^l = M_G^d = 0$;

* Napadni momenti u karakterističnim tačkama:

$M_A = 0$

$M_N = F_A \cdot 1 = 2 \text{ kNm}$

$M_D = 3F_A - 2 \cdot 2 \cdot 1 = 4 \text{ kNm}$

$M_G = F_A \cdot 5 - 2 \cdot 4 \cdot 2 - F_1 \cdot 2 = 10 - 8 - 2 = 0$

$M_B = -F_G \cdot 1 = -3 \text{ kNm}$

$M_E = -3F_G + 2Y_B = 1 \text{ kNm}$

$M_C = M_K = \mathfrak{M}_K = F_3 \cdot 1 = 1 \text{ kNm}$

1. POLJE $\overline{AN}$: $0 \leq x \leq 1 \text{ m}$

$M(x) = F_A \cdot x = 2x$

$M(0) = 0$

$M(1) = M_N = 2 \text{ kNm}$

$F_T(x) = \frac{dM(x)}{dx} = F_A = 2 \text{ kN}$

2. POLJE $\overline{ND}$: $0 \leq x \leq 2 \text{ m}$

$M(x) = F_A(1+x) - 2x\frac{x}{2} =$

$= F_A + F_A x - \frac{1}{2} 2x^2 =$

$= 2 + 2x - \frac{1}{2}x^2$

$M(0) = M_N = 2 \text{ kNm}$

$M(1) = 3,5 \text{ kNm}$

$M(2) = M_D = 4 \text{ kNm}$

$\frac{dM(x)}{dx} = F_T(x) = F_A - 2x = 2 - x$

$\frac{dM(x)}{dx} = 0 \Rightarrow 2 - x = 0 \Rightarrow x_e = 2 \text{ m}$

$M(x_e) = M_D = 4 \text{ kNm}$

$F_T(0) = 2 \text{ kN}$

$F_T(2) = 0$

3. POLJE $\overline{DG}$: $0 \leq x \leq 2 \text{ m}$

$M(x) = F_G \cdot x - 2x\frac{x}{2} = F_G \cdot x - \frac{1}{2}2x^2 =$

$= 3x - \frac{1}{2}x^2$

$M(0) = M_G = 0$

$M(1) = 2,5 \text{ kNm}$

$M(2) = M_D = 4 \text{ kNm}$

$F_T(x) = -\frac{M(x)}{dx} = -3 + x$

$F_T(0) = -3 \text{ kN}$

$F_T(2) = -1 \text{ kN}$

4. POLJE $\overline{GB}$: $0 \leq x \leq 1 \text{ m}$

$M(x) = -F_G \cdot x = -3x$

$M(0) = M_G = 0$

$M(1) = M_B = -3 \text{ kNm}$

$F_T(x) = \frac{dM(x)}{dx} = -3 \text{ kN}$

5. POLJE $\overline{BE}$: $0 \leq x \leq 2 \text{ m}$

$M(x) = -F_G(1+x) + Y_B \cdot x = -F_G - F_G \cdot x + Y_B x =$

$= -3 - 3x + 5x = 2x - 3$

$M(0) = M_B = -3 \text{ kNm}$

$M(2) = 1 \text{ kNm}$

$F_T(x) = \frac{dM(x)}{dx} = 2 \text{ kN}$

$F_N(x) = -X_B = -1 \text{ kN}$

6. POLJE $\overline{EC}$: $0 \leq x \leq 2 \text{ m}$

$M(x) = \mathfrak{M}_K + F_C \cdot x = 1 + 0 \cdot x = 1 \text{ kNm}$

$F_T(x) = -\frac{dM(x)}{dx} = 0$

$F_N(x) = -F_3 = -1 \text{ kN}$

7. POLJE $\overline{CK}$: $0 \leq x \leq 1 \text{ m}$

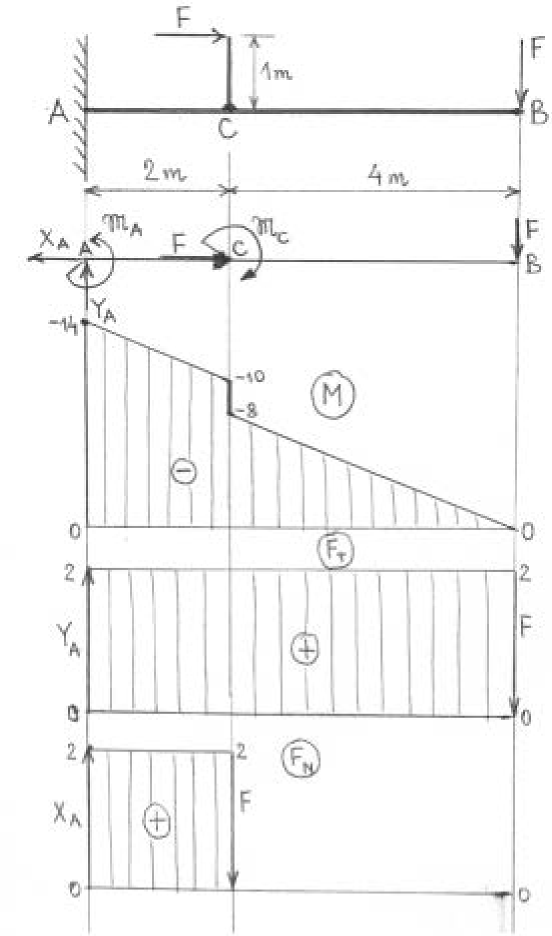
$M(x) = \mathfrak{M}_K = 1 \text{ kNm}$

$F_T(x) = 0$

$F_N(x) = -F_3 = -1 \text{ kN}$

ZADATAK BR. 33

Odrediti otpore uklještenja i nacrtati statičke dijagrame lakog konzolnog nosača $\overline{AB}$ opterećenog koncentrisanom silom F na slobodnom kraju B i horizontalnom ekscentričnom silom F = 2 kN kao što je prikazano na slici.



$\mathfrak{M}_C = F\cdot 1 = 2$ kNm

1) $\sum X_i = 0$: $F - X_A = 0 \Rightarrow X_A = F = 2$ kN

2) $\sum Y_i = 0$: $Y_A - F = 0 \Rightarrow Y_A = F = 2$ kN

3) $\sum M_A = 0$: $-F\cdot 6 - \mathfrak{M}_C + \mathfrak{M}_A = 0$

$\mathfrak{M}_A = 6F + \mathfrak{M}_C = 12 + 2$

$\mathfrak{M}_A = 14$ kNm

$M_A = -\mathfrak{M}_A = -14$ kNm

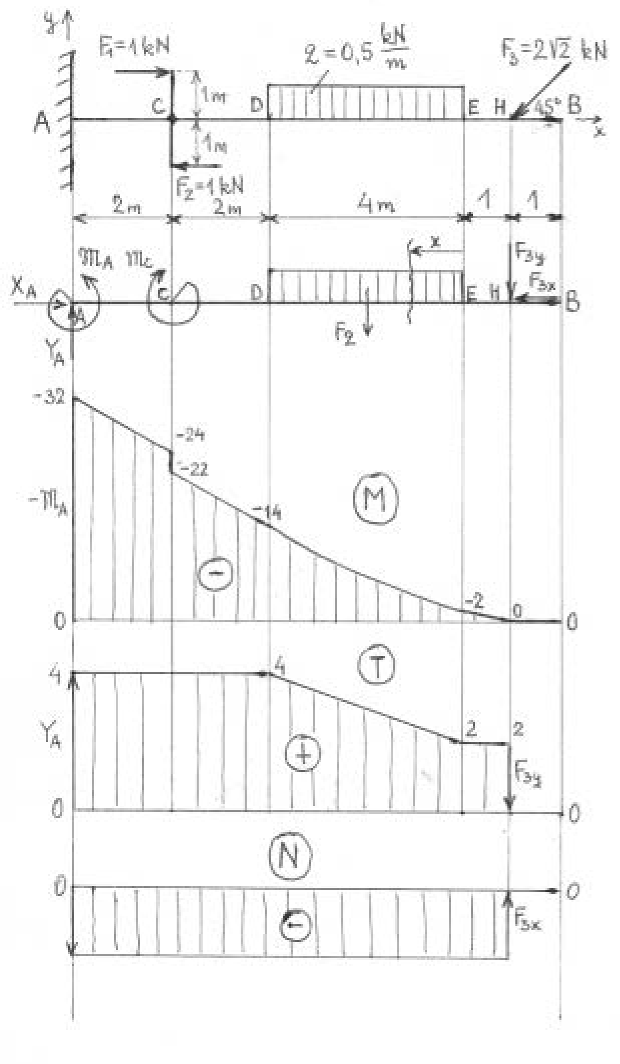
$M_{C,l} = Y_A\cdot 2 - \mathfrak{M}_A = -10$ kNm

$M_{C,d} = Y_A\cdot 2 - \mathfrak{M}_A + \mathfrak{M}_C = -8$ kNm

$M_B = 0$

ZADATAK BR. 34

Za laki konzolni nosač, dimenzija i opterećenja datih na slici, odrediti otpor ukliještenja i nacrtati statičke dijagrame.



1) $\sum X_i = 0: X_A - F_{3x} = 0$; $F_{3x} = F_3 \cos 45° = 2$ kN

$X_A = F_{3x} = 2$ kN

2) $\sum Y_i = 0: Y_A - F_q - F_{3y} = 0$; $F_q = q \cdot 4 = 2$ kN

$F_{3y} = F_3 \sin 45° = 2$ kN

$Y_A = F_q + F_{3y}$

$Y_A = 4$ kN

3) $\sum M_A = 0: -F_{3y} \cdot 9 - F_q \cdot 6 - \mathfrak{M}_C + \mathfrak{M}_A = 0$;

$\mathfrak{M}_C = F_1 \cdot 2 = F_2 \cdot 2 = 2$ kNm

$\mathfrak{M}_A = \mathfrak{M}_C + 6F_q + 9F_{3y}$

$\mathfrak{M}_A = 2 + 12 + 18$

$\mathfrak{M}_A = 32$ kNm

$M_A = -\mathfrak{M}_A = -32$ kNm

$M_{C,l} = -\mathfrak{M}_A + Y_A \cdot 2 = -32 + 8 = -24$ kNm

$M_{C,d} = -\mathfrak{M}_A + Y_A \cdot 2 + \mathfrak{M}_C = -22$ kNm , ili :

$M_{C,d} = -F_{3y} \cdot 7 - F_q \cdot 4 = -14 - 8 = -22$ kNm

$M_D = -F_{3y} \cdot 5 - F_q \cdot 2 = -14$ kNm

$M_E = -F_{3y} \cdot 1 = -2$ kNm

$M_H = 0$

$M_B = 0$

$0 \le x \le 4\,m$:

$M(x) = -F_{3y}(1+x) - qx\frac{1}{2}x = -2 - 2x - \frac{1}{4}x^2$

$M(0) = M_E = -2$ kNm

$M(4) = M_D = -14$ kNm

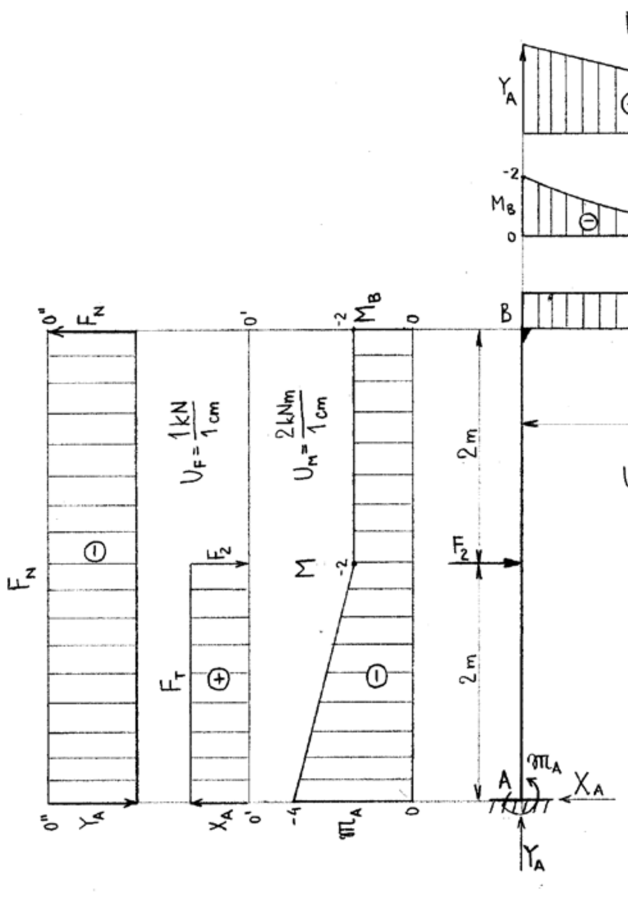
$\frac{dM}{dx} = -2 - \frac{1}{2}x_e$; $-2 - \frac{1}{2}x_e = 0 \Rightarrow x_e = -4\,m \notin [0,4]$

$F_T(x) = -\frac{dM(x)}{dx} = 2 + \frac{1}{2}x$, $0 \le x \le 4\,m$

ZADATAK BR. 35

Konzola AB savijena je kod B pod pravim uglom i opterećena datim teretima. Odrediti otpore oslonaca i nacrtati statičke dijagrame ako su:

$q = 0{,}5$ kN/m ; $F_1 = 0{,}5$ kN ; $F_2 = 1$ kN



1) $\Sigma X_i = 0$: $F_2 - X_A = 0 \Rightarrow X_A = 1$ [kN]

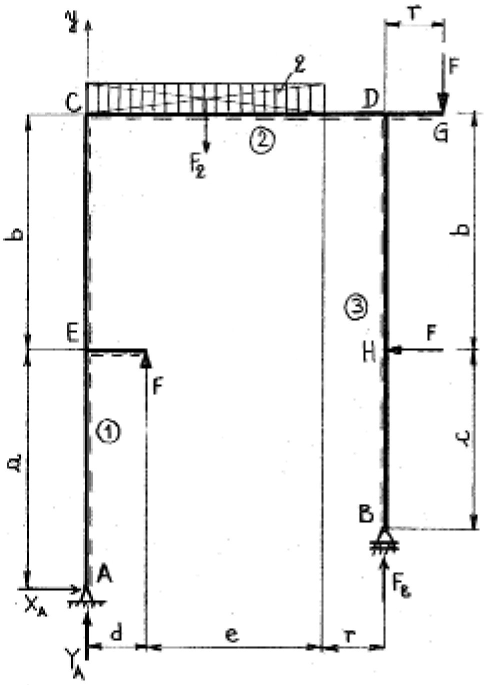
2) $\Sigma Y_i = 0$: $Y_A - F_2 - F_1 = 0 \Rightarrow Y_A = 1{,}5$ [kN]

3) $\Sigma M_A = 0$: $\mathfrak{M}_A - 2F_2 - 1 \cdot F_2 - 2F_1 = 0 \Rightarrow \mathfrak{M}_A = 4$ [kNm]

$M_B = -2F_1 - 1F_2 = -2$ [kNm]

ZADATAK BR. 36

Za ramovski nosač, oblika i dimenzija prikazanih na slici, odrediti analitički otpore oslonaca.



$a = 4\ [m]$
$b = 4\ [m]$
$c = 3\ [m]$
$d = 1\ [m]$
$e = 3\ [m]$
$r = 1\ [m]$
$F = 60\ [kN]$
$q = 10\ [kN/m]$

$$U_L = \frac{1\ [m]}{1\ [cm]}$$

$$F_q = q(d+e) = 40\ [kN]$$

Određivanje otpora oslonaca analitičkom metodom:

1) $\sum X = 0 : \quad X_A - F = 0 \Rightarrow \underline{X_A = F = 60\ [kN]}$

2) $\sum Y = 0 : \quad Y_A + \cancel{F} - F_q - \cancel{F} + F_B = 0 \Rightarrow Y_A + F_B = F_q = 40\ [kN]$

3) $\sum \overset{\frown}{M}_A = 0 : \quad -F \cdot d + F_q\left(\frac{d+e}{2}\right) + F(d+e+2r) - F \cdot a - F_B(d+e+r) = 0$

$$F_B = \frac{1}{(1+3+1)}\left[-60 \cdot 1 + 40\left(\frac{1+3}{2}\right) + 60(1+3+2 \cdot 1) - 60 \cdot 4\right]$$

$$F_B = \frac{1}{5}\left[-60 + 80 + 360 - 240\right] = \frac{140}{5} \Rightarrow \underline{F_B = 28\ [kN]}$$

2) $\Rightarrow Y_A = F_q - F_B = 40 - 28 \Rightarrow \underline{Y_A = 12\ [kN]}$

$$F_A = \sqrt{X_A^2 + Y_A^2} = \sqrt{60^2 + 12^2} \Rightarrow \underline{F_A = 61{,}1\ [kN]}$$

$$tg\,\alpha = \frac{Y_A}{X_A} = \frac{12}{60} = 0{,}2\ ; \quad \alpha = arctg\,0{,}2 \Rightarrow \alpha = 11{,}3\ [^\circ]$$

Jednačina provere:

$$\sum \overset{\frown}{M}_B = -X_A(a-c) + Y_A(d+e+r) + F(e+r) - F_q\left(r + \frac{d+e}{2}\right) + F \cdot r - F \cdot c =$$
$$= -\cancel{60 \cdot 1} + 12 \cdot 5 + 60 \cdot 4 - 40 \cdot 3 + \cancel{60 \cdot 1} - 60 \cdot 3 = 0$$

TEŽIŠTA

Težište tela je ona njegova tačka koja pri ma kojem položaju tela ostaje nepromenjena, tj. ostaje napadna tačka njegove težine.

1. TEŽIŠTA LINIJA - Kada su dve dimenzije tela male prema trećoj, tako da ih možemo zanemariti, telo smatramo teškom linijom.

$$x_c = \frac{\sum_{i=1}^{n} L_i x_i}{L} \; ; \quad y_c = \frac{\sum_{i=1}^{n} L_i y_i}{L} \; ; \quad z_c = \frac{\sum_{i=1}^{n} L_i z_i}{L}$$

- ako se homogeno telo može podeliti na konačan broj linija

$$x_c = \frac{\int x\,dl}{L} \; ; \quad y_c = \frac{\int y\,dl}{L} \; ; \quad z_c = \frac{\int z\,dl}{L}$$

- ako telo ne možemo podeliti na konačan broj linija

2. TEŽIŠTA POVRŠINA - Kada je jedna dimenzija tela mala prema drugim dvema, tako da je možemo zanemariti, telo smatramo teškom površinom.

$$x_c = \frac{\sum_{i=1}^{n} A_i x_i}{A} \; ; \quad y_c = \frac{\sum_{i=1}^{n} A_i y_i}{A} \; ; \quad z_c = \frac{\sum_{i=1}^{n} A_i z_i}{A} \; ; \quad A = \sum_{i=1}^{n} A_i$$

$$x_c = \frac{\iint_A x\,dA}{A} \; ; \quad y_c = \frac{\iint_A y\,dA}{A} \; ; \quad z_c = \frac{\iint_A z\,dA}{A} \; ; \quad A = \iint_{(A)} dA$$

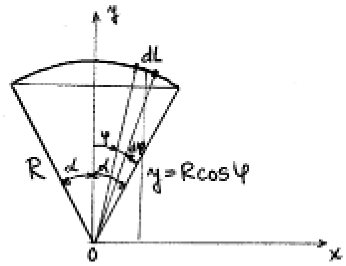
3. TEŽIŠTA ZAPREMINA

$$x_c = \frac{\iiint_V x\,dV}{V} \; ; \quad y_c = \frac{\iiint_V y\,dV}{V} \; ; \quad z_c = \frac{\iiint_V z\,dV}{V}$$

$$x_c = \frac{\sum_{i=1}^{n} x_i V_i}{V} \; ; \quad y_c = \frac{\sum_{i=1}^{n} y_i V_i}{V} \; ; \quad z_c = \frac{\sum_{i=1}^{n} z_i V_i}{V}$$

PRIMER T1

Odrediti koordinate težišta kružnog luka.



$$z_c = 0 \; ; \quad x_c = 0$$

$$y_c = \frac{\int_L y\,dL}{\int_L dL}$$

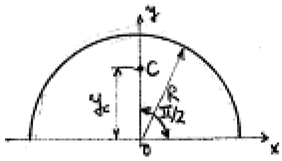
$$dL = R\,d\varphi$$

$$L = \int_L dL = \int_{-\alpha}^{\alpha} R\,d\varphi = R\int_{-\alpha}^{\alpha} d\varphi = R\varphi\Big|_{-\alpha}^{\alpha} = R\left[\alpha-(-\alpha)\right]$$

$$L = 2R\alpha$$

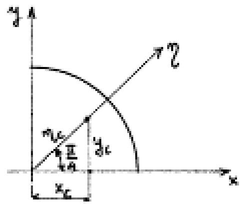
$$\int_L y\,dL = \int_{-\alpha}^{\alpha} R\cos\varphi \cdot R\,d\varphi = R^2\int_{-\alpha}^{\alpha}\cos\varphi\,d\varphi = R^2\sin\varphi\Big|_{-\alpha}^{\alpha} = R^2\left[\sin\alpha - \sin(-\alpha)\right] = 2R^2\sin\alpha$$

$$y_c = \frac{2R^2\sin\alpha}{2R\alpha} \Rightarrow y_c = \frac{R\sin\alpha}{\alpha} \quad (\alpha \text{ u radijanima})$$



$$y_c = \frac{R\sin\frac{\pi}{2}}{\frac{\pi}{2}} = \frac{R\cdot 1}{\frac{\pi}{2}} \Rightarrow y_c = \frac{2R}{\pi}$$

$$x_c = 0$$



$$\alpha = \frac{\pi}{4}$$

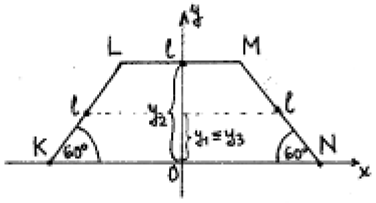
$$x_c = y_c = \eta_c\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}\eta_c$$

$$\eta_c = \frac{R\sin\frac{\pi}{4}}{\frac{\pi}{4}} = \frac{R\frac{\sqrt{2}}{2}}{\frac{\pi}{4}} \Rightarrow \eta_c = \frac{2\sqrt{2}\,R}{\pi}$$

$$x_c = y_c = \frac{\sqrt{2}}{\cancel{2}}\,\frac{\cancel{2}\sqrt{2}\,R}{\pi} \Rightarrow x_c = y_c = \frac{2R}{\pi}$$

PRIMER T2

Odrediti položaj težišta poligona KLMN, strane $\ell = 6\,cm$, koji predstavlja polovinu šestougaonika.



$$\underline{x_c = 0}$$

$$y_c = \frac{\sum L_i y_i}{L}$$

$$L = \sum_{i=1}^{3} L_i = 3\ell = 18\ cm$$

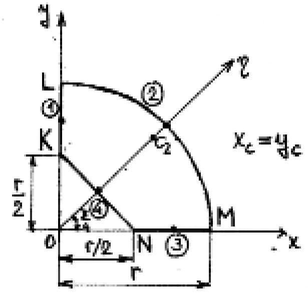
$$y_1 = y_3 = \frac{\ell}{2}\sin 60^\circ = \frac{\ell}{2}\,\frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2}\ cm$$

$$y_2 = \ell\cdot\sin 60^\circ = \ell\frac{\sqrt{3}}{2} = 3\sqrt{3}\ cm$$

$$y_c = \frac{\ell\cdot y_1 + \ell\cdot y_2 + \ell\cdot y_3}{L} = \frac{\ell(2\cdot y_1 + y_2)}{L} = \frac{6\left(\cancel{2}\cdot\frac{3\sqrt{3}}{\cancel{2}} + 3\sqrt{3}\right)}{18} \Rightarrow \underline{y_c = 2\sqrt{3}\ cm}$$

PRIMER T3

Odrediti koordinate težišta homogenog poligona KLMN.



$$x_c = y_c$$

ℓ	L_i [cm]	y_i [cm]	$L_i y_i$ [cm²]
1	$r/2$	$\frac{3}{4}r$	$\frac{3}{8}r^2$
2	$\pi r/2$	$\frac{2r}{\pi}$	r^2
3	$r/2$	0	0
4	$r\frac{\sqrt{2}}{2}$	$\frac{r}{4}$	$\frac{\sqrt{2}}{8}r^2$
Σ	$r\left(\frac{1}{2}+\frac{\pi}{2}+\frac{1}{2}+\frac{\sqrt{2}}{2}\right)$	/	$r^2\left(\frac{3}{8}+1+\frac{\sqrt{2}}{8}\right)$

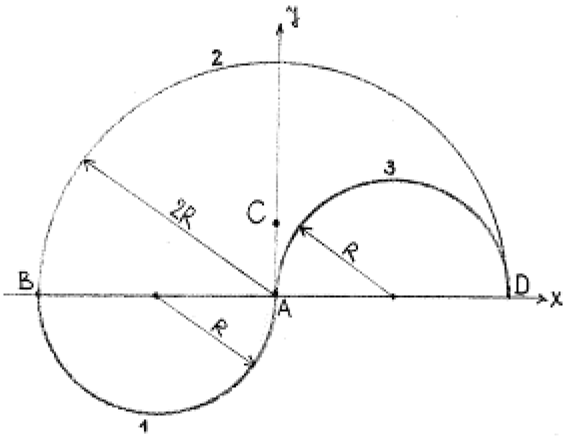
$$r_2 = \frac{r\sin\alpha}{\alpha} = \frac{r\frac{\sqrt{2}}{2}}{\frac{\pi}{4}} = \frac{2\sqrt{2}\,r}{\pi}$$

$$y_2 = r_2\cos\frac{\pi}{4} = \frac{2\sqrt{2}\,r}{\pi}\,\frac{\sqrt{2}}{2} \Rightarrow y_2 = \frac{2r}{\pi}$$

$$y_c = x_c = \frac{\sum_{i=1}^{4} L_i y_i}{\sum_{i=1}^{4} L_i} = \frac{\frac{r^2}{8}(11+\sqrt{2})}{\frac{r}{2}(2+\sqrt{2}+\pi)} = \frac{r}{4}\,\frac{(11+\sqrt{2})}{(2+\sqrt{2}+\pi)} \approx 0{,}47\,r$$

PRIMER T4

Analitički odrediti položaj težišta zatvorene kružne linije ABDA, pri čemu je $R = 4$ cm.



	i	L_i	x_i	y_i	$L_i x_i$	$L_i y_i$
$\widehat{AB}$	1	$R\pi$	$-R$	$-\frac{2R}{\pi}$	$-R^2\pi$	$-2R^2$
$\widehat{BD}$	2	$2R\pi$	0	$\frac{4R}{\pi}$	0	$8R^2$
$\widehat{DA}$	3	$R\pi$	R	$\frac{2R}{\pi}$	$R^2\pi$	$2R^2$
	Σ	$4R\pi$	/	/	0	$8R^2$

$$y_1 = -\frac{R\sin\frac{\pi}{2}}{\frac{\pi}{2}} = -\frac{2R}{\pi}$$

$$y_2 = \frac{2R\sin\frac{\pi}{2}}{\frac{\pi}{2}} = \frac{4R}{\pi}$$

$$y_3 = \frac{R\sin\frac{\pi}{2}}{\frac{\pi}{2}} = \frac{2R}{\pi}$$

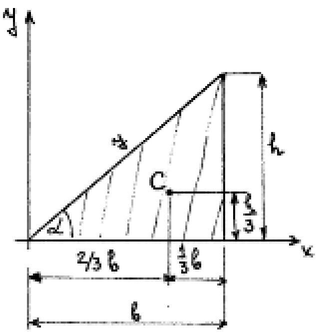
$$x_c = \frac{\sum_{1}^{3} L_i x_i}{\Sigma L_i} = 0$$

$$y_c = \frac{\sum_{i=1}^{3} L_i y_i}{\Sigma L_i} = \frac{8R^{\cancel{2}}}{4\cancel{R}\pi} = \frac{8\cdot\cancel{4}}{\cancel{4}\cdot\pi} \Rightarrow y_c = 2{,}54 \text{ cm}$$

$C\,(0;\ 2{,}54)\ [\text{cm}]$

PRIMER T5

Odrediti težište površine omeđene pravom koja prolazi kroz koordinatni početak.



$y = kx \;;\; k = \operatorname{tg}\alpha = \frac{h}{b}$

$y = f(x) = \frac{h}{b}x$

$A = \int_{x_1}^{x_2} f(x)\,dx = \int_0^b \frac{h}{b}x\,dx = \frac{h}{b}\int_0^b x\,dx = \frac{h}{b}\frac{x^2}{2}\Big|_0^b$

$A = \frac{h}{\cancel{b}}\frac{b^{\cancel{2}}}{2} \Rightarrow A = \frac{1}{2}hb$

$\int_{x_1}^{x_2} x f(x)\,dx = \int_0^b x\frac{h}{b}x\,dx = \frac{h}{b}\int_0^b x^2dx = \frac{h}{b}\frac{x^3}{3}\Big|_0^b = \frac{1}{3}b^2h$

$x_c = \frac{\frac{1}{3}b^2h}{\frac{1}{2}bh} \Rightarrow x_c = \frac{2}{3}b \quad ; \quad x_c = \frac{\int_{x_1}^{x_2} x f(x)\,dx}{\int_{x_1}^{x_2} f(x)\,dx}$

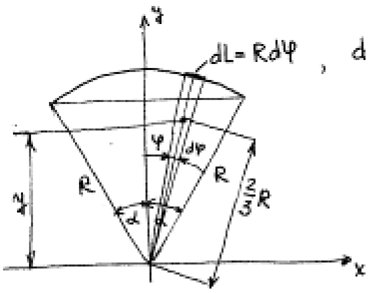
$y_c = \frac{\frac{1}{2}\int_{x_1}^{x_2}[f(x)]^2dx}{\int_{x_1}^{x_2} f(x)\,dx}$

$\frac{1}{2}\int_{x_1}^{x_2}[f(x)]^2dx = \frac{1}{2}\int_0^b\left(\frac{h}{b}x\right)^2dx = \frac{1}{2}\frac{h^2}{b^2}\int_0^b x^2dx = \frac{1}{2}\frac{h^2}{b^2}\frac{x^3}{3}\Big|_0^b = \frac{1}{2}\frac{h^2}{b^2}\frac{b^3}{3} = \frac{1}{6}bh^2$

$y_c = \frac{\frac{1}{6}bh^2}{\frac{1}{2}bh} \Rightarrow y_c = \frac{1}{3}h$

PRIMER T6

Odrediti težište kružnog isečka.



$dL = R\,d\varphi \quad , \quad dA = \frac{1}{2} R\,dL = \frac{1}{2} R^2 d\varphi$

$$y_c = \frac{\int_A y\,dA}{\int_A dA}$$

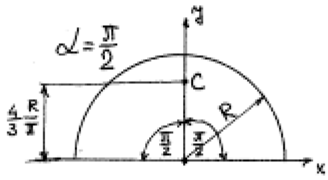
$$A = \int dA = \int \frac{1}{2} R^2 d\varphi = \frac{1}{2} R^2 \int_{-\alpha}^{\alpha} d\varphi = \frac{1}{2} R^2 \varphi \Big|_{-\alpha}^{\alpha} = \frac{1}{2} R^2 2\alpha \quad ; \quad A = R^2 \alpha$$

$$y = \frac{2}{3} R \cos\varphi$$

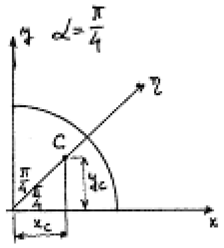
$$\int_A y\,dA = \int \frac{2}{3} R \cos\varphi \cdot \frac{1}{2} R^2 d\varphi = \frac{1}{3} R^3 \int_{-\alpha}^{\alpha} \cos\varphi\, d\varphi = \frac{1}{3} R^3 \sin\varphi \Big|_{-\alpha}^{\alpha} = \frac{1}{3} R^3 2 \sin\alpha = \frac{2}{3} R^3 \sin\alpha$$

$$y_c = \frac{\frac{2}{3} R^{\cancel{3}} \sin\alpha}{\cancel{R^2} \alpha}$$

$$y_c = \frac{2}{3} \frac{R \sin\alpha}{\alpha} \; ; \quad \alpha\,[rad]$$



$$y_c = \frac{2}{3} \frac{R \sin\frac{\pi}{2}}{\frac{\pi}{2}} \Rightarrow y_c = \frac{4}{3} \frac{R}{\pi}$$



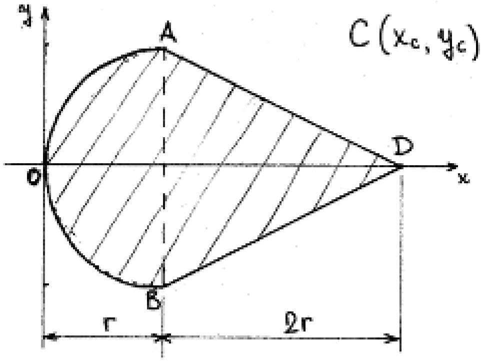
$$\eta_c = \frac{2}{3} \frac{R \sin\frac{\pi}{4}}{\frac{\pi}{4}} = \frac{2}{3} \frac{R \frac{\sqrt{2}}{2}}{\frac{\pi}{4}} = \frac{4\sqrt{2}}{3\pi} R$$

$$x_c = y_c = \eta_c \cos\frac{\pi}{4} = \eta_c \frac{\sqrt{2}}{2} = \frac{4\cancel{\sqrt{2}}}{3\pi} \frac{\cancel{\sqrt{2}}}{\cancel{2}} R$$

$$x_c = y_c = \frac{4}{3} \frac{R}{\pi}$$

PRIMER T7

Odrediti položaj težišta C površine ograničene polukrugom AOB, polupréčnika R, i dvema dužima $\overline{AD}$ i $\overline{BD}$ jednakih dužina.



$C(x_c, y_c)$; $y_c = 0$, $x_c = ?$

$$A = A_1 + A_2 = \frac{\pi r^2}{2} + \frac{1}{2} 2r \cdot 2r = \frac{\pi r^2}{2} + 2r^2 = \frac{r^2}{2}(\pi + 4)$$

$$\sum A_i x_i = \frac{\pi r^2}{2}\left(r - \frac{4r}{3\pi}\right) + 2r^2\left(r + \frac{2r}{3}\right) = r^3\left(\frac{\pi}{2} - \frac{2}{3} + 2 + \frac{4}{3}\right) = r^3\left(\frac{\pi}{2} + \frac{8}{3}\right) =$$

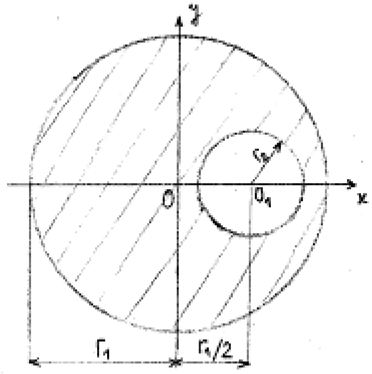
$$= \frac{r^3}{6}(3\pi + 16)$$

$$x_c = \frac{\sum A_i x_i}{A} = \frac{\frac{r^3}{6}(3\pi + 16)}{\frac{r^2}{2}(\pi + 4)} = \frac{r}{3} \cdot \frac{3\pi + 16}{\pi + 4}$$

$x_c \approx 1{,}19\, r$

PRIMER T8

Iz homogenog diska poluprečnika r_1, isečen je kružni otvor poluprečnika r_2. Središte O_1 otvora nalazi se na rastojanju $r_1/2$ od središta O diska. Odrediti položaj težišta diska sa tim kružnim otvorom.



$y_C = 0$

$$x_C = \frac{A_1x_1 - A_2x_2}{A_1 - A_2}$$

$A_1 = r_1^2\pi \quad , \quad x_1 = 0 \quad ; \quad A_1x_1 = 0$

$A_2 = r_2^2\pi \quad , \quad x_2 = \frac{r_1}{2} \quad ; \quad A_2x_2 = \frac{r_1r_2^2\pi}{2}$

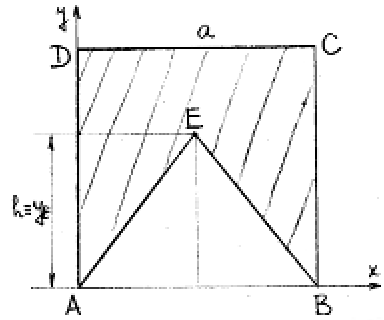
$A_1 - A_2 = \pi(r_1^2 - r_2^2)$

$$x_C = -\frac{r_1r_2^2\pi}{2(r_1^2-r_2^2)\pi} = -\frac{r_1r_2^2}{2(r_1^2-r_2^2)}$$

$$C\left(-\frac{r_1r_2^2}{2(r_1^2-r_2^2)} \; ; \; 0\right)$$

ZADATAK BR. 37

Iz homogene kvadratne ploče ABCD treba iseći jednakokraki trougao ABE, tako da tačka E bude težište preostale površine. Odrediti koordinate tačke E.



$$x_E = \frac{a}{2}$$

$$y_E = \frac{A_1y_1 - A_2y_2}{A_1 - A_2}$$

$A_1 = a^2 \quad , \quad y_1 = \frac{a}{2} \quad ; \quad A_1y_1 = \frac{a^3}{2}$

$A_2 = \frac{1}{2}ay_E \quad ; \quad y_2 = \frac{1}{3}y_E \quad ; \quad A_2y_2 = \frac{ay_E^2}{6}$

$$A_1 - A_2 = a^2 - \frac{1}{2}ay_E$$

$$y_E = \frac{\frac{a^3}{2} - \frac{ay_E^2}{6}}{a^2 - \frac{1}{2}ay_E} = \frac{\frac{3a^2 - y_E^2}{6}}{\frac{6a - 3y_E}{6}} = \frac{3a^2 - y_E^2}{6a - 3y_E}$$

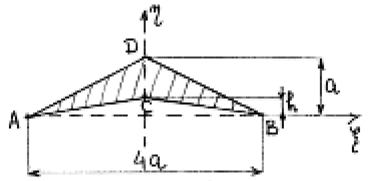
$$6ay_E - 3y_E^2 = 3a^2 - y_E^2 \Rightarrow 2y_E^2 - 6ay_E + 3a^2 = 0$$

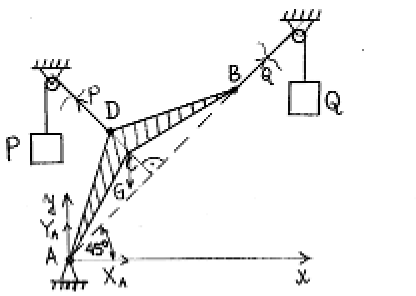
$$y_{E1/2} = \frac{6a \pm \sqrt{36a^2 - 24a^2}}{4} = \frac{6a \pm 2\sqrt{3}a}{4} = \frac{3 \pm \sqrt{3}}{2}a$$

$$y_E = \frac{3-\sqrt{3}}{2}a \approx 0{,}63\,a$$

ZADATAK BR. 38

Homogena pločica ACBD, težine G, sa težištem u tački C, dobijena je isecanjem trougaone pločice ACB iz trougaone pločice ABD. Teme A pločice vezano je zglobno, a za teme B vezano je lako nerastegljivo uže o čiji drugi kraj visi teret $Q = G/2$. Uže je prebačeno preko malog kotura H tako da sa horizontalom gradi ugao od 45° za ravnotežni položaj pločice koji je ostvaren pomoću tega P. Odrediti težinu tega P, koji je vezan za uže DE prebačeno preko malog kotura K, tako da uže ima pravac CD. Odrediti silu u zglobu A.





$C(\xi_c, \eta_c)$

$\xi_c = 0$

$$\eta_c = h = \frac{A_1\eta_1 - A_2\eta_2}{A_1 - A_2}$$

$$A_1 = \frac{1}{2}4a\cdot a = 2a^2; \quad \eta_1 = \frac{1}{3}a$$

$$A_2 = \frac{1}{2}4a\cdot h = 2ah; \quad \eta_2 = \frac{1}{3}h$$

$$h = \frac{2a^2\cdot\frac{1}{3}a - 2ah\frac{1}{3}h}{2a^2 - 2ah}$$

$$2ha^2 - 2h^2a = \frac{2}{3}a^3 - \frac{2}{3}h^2a \;/:a$$

$$2h^2 - \frac{2}{3}h^2 - 2ah + \frac{2}{3}a^2 = 0$$

$$\frac{4}{3}h^2 - 2ah + \frac{2}{3}a^2 = 0 \;/\cdot 3$$

$$4h^2 - 6a\cdot h + 2a^2 = 0$$

$$h_{1/2} = \frac{6a \pm \sqrt{36a^2 - 32a^2}}{8} = \frac{6a \pm 2a}{8} = \frac{3a \pm a}{4}$$

$$\cancel{h_1 = a}, \quad h_2 = \frac{1}{2}a \;\Rightarrow\; h = \frac{1}{2}a$$

1) $\sum X_i = 0: \quad X_A - P\cos 45° + Q\cos 45° = 0 \;\Rightarrow\; X_A = (P - Q)\frac{\sqrt{2}}{2}$

2) $\sum Y_i = 0: \quad Y_A + P\sin 45° + Q\sin 45° - G = 0 \;\Rightarrow\; Y_A = G - (P + Q)\frac{\sqrt{2}}{2}$

3) $\sum M_A = 0: \quad P\cdot 2a - G\cdot x_G = 0$

$$x_G = 2a\cdot\cos 45° - \frac{1}{2}a\cos 45° = 2a\frac{\sqrt{2}}{2} - \frac{1}{2}a\frac{\sqrt{2}}{2} = a\frac{\sqrt{2}}{2}\left(2 - \frac{1}{2}\right) = \frac{3\sqrt{2}}{4}a$$

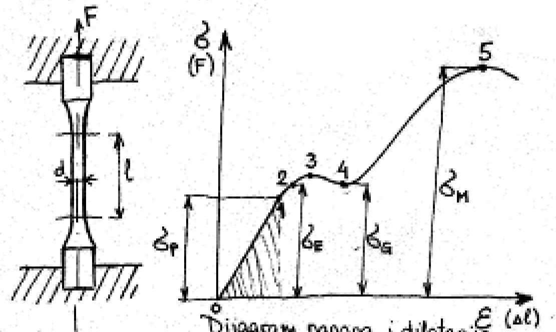
$$P\cdot 2\cancel{a} - G\frac{3\sqrt{2}}{4}\cancel{a} = 0 \;\Rightarrow\; P = G\frac{3\sqrt{2}}{8}$$

1) $\Rightarrow X_A = \left(G\frac{3\sqrt{2}}{8} - G\frac{1}{2}\right)\frac{\sqrt{2}}{2} = G\left(\frac{3\sqrt{2}}{8} - \frac{1}{2}\right)\frac{\sqrt{2}}{2} = G\frac{6 - 4\sqrt{2}}{16}$

2) $\Rightarrow Y_A = G - G\left(\frac{3\sqrt{2}}{8} + \frac{1}{2}\right)\frac{\sqrt{2}}{2} = G\frac{10 - \sqrt{2}}{16}$

OTPORNOST MATERIJALA

1. AKSIJALNO NAPREZANJE



Dijagram napona i dilatacije (sile i izduženja)

1 - granica proporcionalnosti
2 - granica elastičnosti
3 - početak razvlačenja
4 - završetak (granica) razvlačenja
5 - jačina (čvrstoća) materijala pri kidanju (istezanju)

σ_P [N/cm²] - napon na granici proporcion.
σ_E - napon na granici elastičnosti
σ_M - čvrstoća materijala na kidanje

$\varepsilon = \frac{\Delta l}{l}$; Δl - izduženje (skraćenje)

dilatacija (uzdužna) - relativno izduženje (skraćenje)

$\Delta l = l_1 - l$; l_1 - dužina posle deformacije
l - prvobitna dužina

$\sigma = \frac{F}{A}$ $\left[\frac{N}{cm^2}\right]$ - napon

$\boxed{\sigma = E \cdot \varepsilon}$ - I oblik Hukovog zakona (zakon proporcionalnosti)
Napon je linearna f-ja dilatacije u oblasti proporcionalnosti za slučaj aksijalnog naprezanja

$$\sigma = \frac{F}{A} = E\frac{\Delta l}{l} \Rightarrow \boxed{\Delta l = \frac{\sigma \cdot l}{E} = \frac{F \cdot l}{E \cdot A} = \frac{F \cdot l}{U}}$$ - II oblik Hukovog zakona

E [N/cm²] - koeficijent proporcionalnosti (modul elastičnosti), Jungov modul
U [N] - krutost štapa (na istezanje)

$$\boxed{l_1 = l + \Delta l = l + l\varepsilon = l(1+\varepsilon)}$$

$\boxed{\varepsilon_p = -\mu \varepsilon}$ - poprečna dilatacija (kontrakcija) štapa
μ - Poasonov koeficijent proporcionalnosti

$$\varepsilon_p = \frac{\Delta A}{A} = \frac{A_1 - A}{A} = \frac{A_1}{A} - 1$$

$\varepsilon_V = \frac{\Delta V}{V}$ - kubna (zapreminska) dilatacija

$$\boxed{\varepsilon_V = (1-2\mu)\varepsilon}$$

$\boxed{\varepsilon_t = \alpha(t_2 - t_1) = \alpha \Delta t}$ - dilatacija usled promene temperature (temperaturna dilatacija)

$$\sigma_t = E \cdot \varepsilon_t = E\alpha(t_2 - t_1)$$

$\Delta l_t = \alpha(t_2 - t_1) \cdot l$; α - termički koeficijent linearnog širenja ; Odgovara izduženju štapa dužine 1 cm pri povećanju temperature za 1°C.

ZADATAK BR. 1

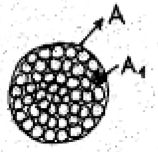
Čelična probna epruveta prečnika $d = 20\ mm$ i dužine $l = 10d$ iskidana je na mašini za kidanje pri najvećem opterećenju od $F_M = 142{,}1\ kN$. Njena dužina posle kidanja iznosi $l_1 = 247\ mm$. Kolika je čvrstoća materijala na kidanje i koliko je izduženje?

$$\sigma_M = \frac{F_M}{A} = \frac{F_M}{\frac{d^2\pi}{4}} = \frac{142{,}1}{\frac{2^2\pi}{4}} = 45{,}25\ \frac{kN}{cm^2}$$

$$\Delta l = l_1 - l = l_1 - 10d = 247 - 200 = 47\ mm$$

ZADATAK BR. 2

Čelično uže dizalice treba da nosi teret od $F = 100\ kN$. Koliko žica prečnika $d = 1{,}13\ mm$ treba da ima uže ako je dozvoljeni napon na istezanje $\sigma_{de} = 25\ kN/cm^2$?



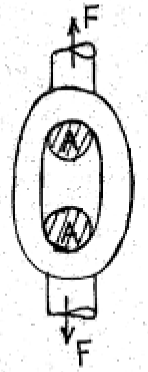
$$\sigma_{de} \geq \sigma = \frac{F}{A} \Rightarrow A \geq \frac{F}{\sigma_{de}} = \frac{100}{25} = 4\ cm^2$$

$$A = n A_1 \quad , \quad A_1 = \frac{d^2\pi}{4} = 1\ mm^2 = 10^{-2}\ cm^2$$

$$n = \frac{A}{A_1} = \frac{4}{10^{-2}} = 400 \text{ komada žica}$$

ZADATAK BR. 3

Karika lanca dizalice prenosi silu $F = 31{,}4\ kN$. Odrediti prečnik kružnog poprečnog preseka karike ako je dozvoljeni napon na istezanje $\sigma_{de} = 20\ kN/cm^2$.



$$\sigma_{de} \geq \sigma = \frac{F}{2A} \Rightarrow A \geq \frac{F}{2\sigma_{de}}$$

$$\frac{d^2\pi}{4} \geq \frac{F}{2\sigma_{de}} \Rightarrow d \geq \sqrt{\frac{2F}{\sigma_{de}\pi}}$$

$$d \geq 1\ cm$$

ZADATAK BR. 4

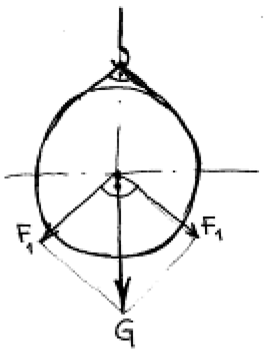
Bakarna žica dužine $l = 1{,}5\,m$ i površine poprečnog preseka $3\,mm^2$ izduži se pod dejstvom sile $F = 20\,N$ za $\Delta l = 0{,}9\,mm$.
Odrediti modul elastičnosti.

$$\Delta l = \frac{F \cdot l}{E \cdot A} \Rightarrow E = \frac{F \cdot l}{A \cdot \Delta l} = \frac{20 \cdot 1{,}5 \cdot 10^2}{0{,}9 \cdot 10^{-1} \cdot 3 \cdot 10^{-2}} = 1{,}11 \cdot 10^6 \frac{N}{cm^2}$$

ZADATAK BR. 5

Kotao težine 29 kN obešen je pomoću lanca na kuku. Krajevi lanca čine ugao 90°.
a) Odrediti sile u delovima lanca;
b) Dimenzionisati kariku lanca kružnog preseka ako je $\sigma_{de} = 6000 \frac{N}{cm^2}$.

$G = 29\,kN$; $\sigma_{de} = 6000 \frac{N}{cm^2}$; $F_1, d = ?$



a) $G = F_1 \sqrt{2}$

$$F_1 = \frac{G}{\sqrt{2}} = 20506\,N$$

b) $\sigma_{de} \geq \frac{F_1}{2A}$

$$A = \frac{d^2 \pi}{4} = \frac{F_1}{2\sigma_{de}}$$

$$d = \sqrt{\frac{2F_1}{\pi \sigma_{de}}} \approx 1{,}5\,cm$$

ZADATAK BR. 6

Horizontalni čelični štap, dužine 4 m, kvadratnog poprečnog preseka $1\,cm^2$, izdužio se pri zagrevanju za 1 mm.
Kolikim silama treba pritiskati na krajevima štapa da bi se izduženje poništilo? $E = 2 \cdot 10^7\,N/cm^2$

$l = 4\,m$, $A = 1\,cm^2$, $\Delta l = 1\,mm$; $F = ?$

$$F = A \cdot \sigma = A \cdot E\varepsilon = A \cdot E \cdot \frac{\Delta l}{l} = 1 \cdot 2 \cdot 10^7 \cdot \frac{0{,}1}{400} = 5000\,N$$

ZADATAK BR. 7

Čelični štap AB obostrano je uklešten pri temperaturi 0 °C.
Odrediti napon u štapu ako je temperatura porasla na 40 °C.

$\alpha = 0{,}0000125$; $E = 2\cdot 10^{7}\ N/cm^{2}$

$t_1 = 0°C$, $t_2 = 40°C \Rightarrow \Delta t = 40°C$

$\sigma = E\cdot\varepsilon_t = E\cdot\alpha\Delta t = 2\cdot 10^{7}\cdot 0{,}0000125\cdot 40 = 10000\ N/cm^{2}$

ZADATAK BR. 8

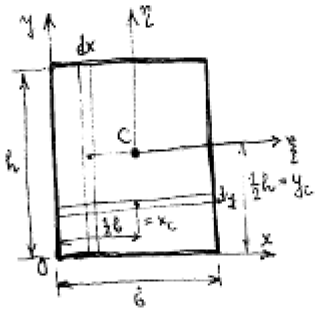
Na kom razmaku treba namestiti, pri temperaturi $t_1 = 10°C$, krajeve šina, dužine 12 m, da bi se pri temperaturi $t_2 = 50°C$ dodirivale ne izazivajući napon? Koliki se napon javlja u šini ako temperatura poraste do $t_3 = 70°C$?

$\alpha = 0{,}0000125$; $E = 2\cdot 10^{7}\ N/cm^{2}$

$\varepsilon_t = \alpha(t_2 - t_1) = \frac{\Delta l}{l} \Rightarrow \Delta l = l\cdot\alpha(t_2 - t_1) = 0{,}6\ cm = 6\ mm$

$\sigma = E\cdot\varepsilon_t' = E\cdot\alpha(t_3 - t_2) = 5000\ N/cm^{2}$

MOMENTI INERCIJE RAVNIH PRESEKA



$$A = b \cdot h$$

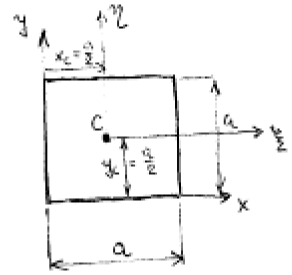
$$I_x = \iint_A y^2 dA = \iint_A y^2 b \cdot dy = b\int_0^h y^2 dy = \frac{b}{3} y^3 \Big|_0^h = \frac{bh^3}{3}$$

$$I_y = \iint_A x^2 dA = \iint_A x^2 h \cdot dx = h\int_0^b x^2 dx = \frac{h}{3} x^3 \Big|_0^b = \frac{hb^3}{3}$$

$$I_o = I_x + I_y = \frac{bh}{3}\left(b^2 + h^2\right)$$

$$I_\xi = I_x - A \cdot y_c^2 = \frac{bh^3}{3} - bh \cdot \left(\tfrac{1}{2}h\right)^2 = \frac{bh^3}{12}$$

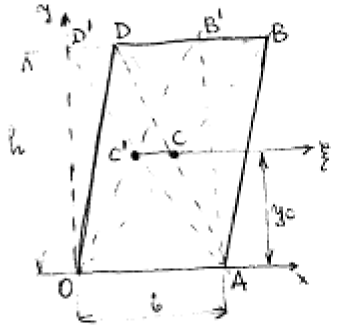
$$I_\eta = I_y - A \cdot x_c^2 = \frac{b^3h}{3} - bh\left(\tfrac{1}{2}b\right)^2 = \frac{hb^3}{12}$$



$$A = a^2$$

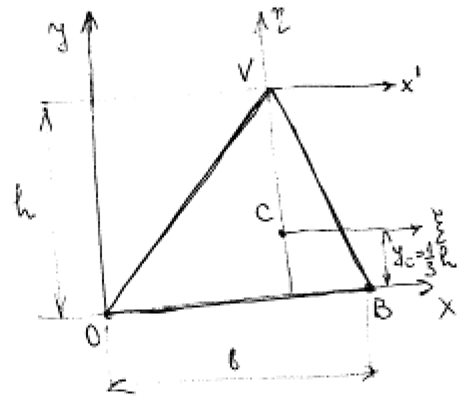
$$I_x = I_y = \frac{a^4}{3}$$

$$I_\xi = I_\eta = \frac{a^4}{12}$$



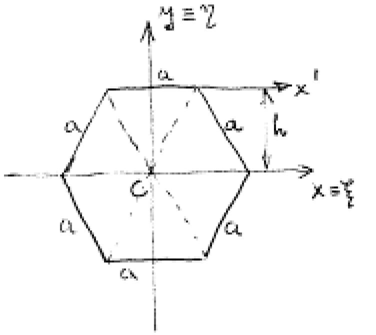
$$I_x = \frac{bh^3}{3}$$

$$I_\xi = \frac{bh^3}{12}$$

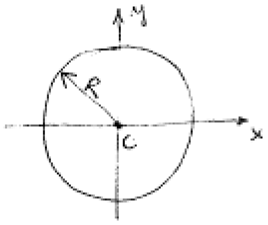


$$I_x = \frac{bh^3}{12} \quad ; \quad I_{x'} = I_\xi + A\, y_c^2 = \frac{bh^3}{36} + \left(\tfrac{1}{2}bh\right)\left(\tfrac{2}{3}h\right)^2 = \frac{bh^3}{4}$$

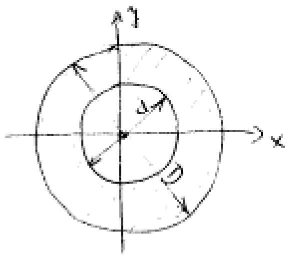
$$I_\xi = \frac{bh^3}{36}$$



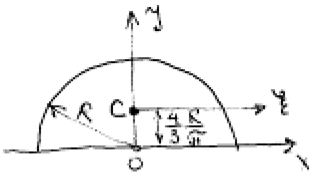
$$I_x = I_\xi = 2\left[2\cdot\frac{bh^3}{12} + 1\cdot\frac{bh^3}{4}\right] = \frac{5}{3}bh^3 \quad ; \quad b = a \;,\; h = a\frac{\sqrt{3}}{2}$$



$$I_x = I_y = \frac{R^4\pi}{4} = \frac{d^4\pi}{64} \quad ; \quad d = 2R$$

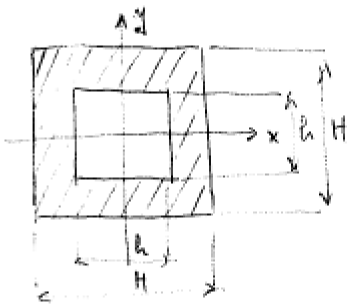


$$I_x = I_y = \frac{(D^4 - d^4)\pi}{64} = \frac{D^4\pi}{64}(1 - \psi^4) \quad ; \quad \psi = \frac{d}{D} = \frac{r}{R}$$

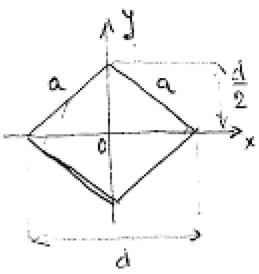


$$I_x = I_y = \frac{R^4\pi}{8}$$

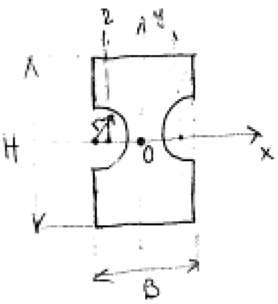
$$I_\xi = I_x - Ay_c^2 = \frac{R^4\pi}{8} - \left(\frac{1}{2}R^2\pi\right)\left(\frac{4}{3}\frac{R}{\pi}\right)^2 = \frac{9\pi^2 - 64}{72\cdot\pi}R^4$$



$$I_x = I_y = \frac{H^4 - h^4}{12}$$

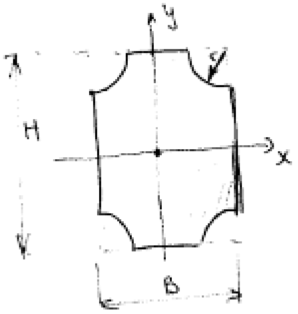


$$I_x = I_y = 2\,\frac{d\cdot\left(\frac{d}{2}\right)^3}{12} = 2\,\frac{(a\sqrt{2})\left(\frac{a\sqrt{2}}{2}\right)^3}{12} = \frac{a^4}{12}$$



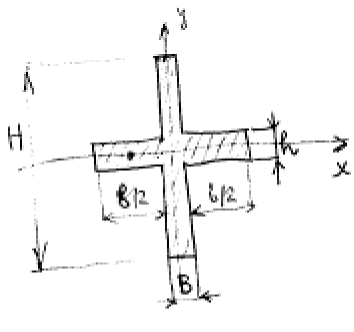
$$I_x = \frac{BH^3}{12} - \frac{r^4\pi}{4}$$

$$I_y = \frac{HB^3}{12} - 2\left[\underbrace{\frac{r^4\pi}{8} - \frac{r^2\pi}{2}\left(\frac{4r}{3\pi}\right)^2}_{I_\eta} + \underbrace{\frac{r^2\pi}{2}\left(\frac{B}{2} - \frac{4}{3}\frac{r}{\pi}\right)^2}_{A\cdot x_c^2}\right]$$



$$I_x = \frac{BH^3}{12} - 2\left[\frac{r^4\pi}{8} - \left(\frac{1}{2}R^2\pi\right)\left(\frac{4}{3}\frac{R}{\pi}\right)^2 + \frac{r^2\pi}{2}\left(\frac{H}{2} - \frac{4}{3}\frac{r}{\pi}\right)^2\right]$$

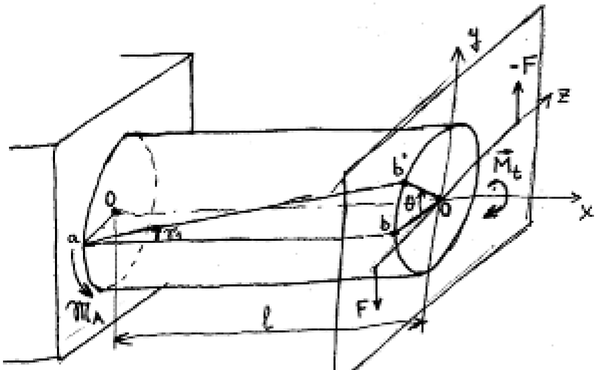
$$I_y = \frac{B^3H}{12} - 2\left[\frac{r^4\pi}{8} - \left(\frac{1}{2}r^2\pi\right)\left(\frac{4}{3}\frac{R}{\pi}\right)^2 + \frac{r^2\pi}{2}\left(\frac{B}{2} - \frac{4}{3}\frac{r}{\pi}\right)^2\right]$$

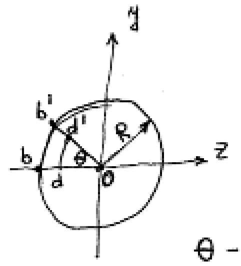


$$I_x = \frac{BH^3}{12} + \frac{bh^3}{12}$$

$$I_y = \frac{HB^3}{12} + 2\left[\frac{h\left(\frac{b}{2}\right)^3}{12} + \left(\frac{b}{2}\cdot h\right)\left(\frac{B}{2} + \frac{b}{4}\right)^2\right]$$

2. UVIJANJE (TORZIJA)





θ - ugao uvijanja

Kod smicanja : $\tau = G \cdot \gamma$; $\tau\ [N/cm^2]$ - tangencijalni napon

$G\ [N/cm^2]$ - modul klizanja

γ - - deformacija (klizanje)

$$\tau = \frac{M_t \cdot r}{I_o}$$; $M_t\ [Ncm]$ - moment uvijanja (torzioni moment)

$$\boxed{\tau_{max} = \frac{M_t \cdot R}{I_o} = \frac{M_t}{W_o}}$$

I glavna j-na uvijanja

I_o - polarni moment inercije poprečnog preseka za tačku O - središte kružnog preseka $[cm^4]$

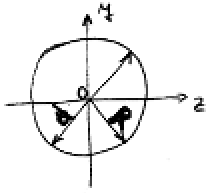
W_o - polarni otporni moment poprečnog preseka za osu O $[cm^3]$

$$W_o = \frac{I_o}{R}$$

$$\boxed{\begin{aligned} \theta\ [rad] &= \frac{l\gamma_1}{R} = \frac{l}{R}\frac{\tau_{max}}{G} = \frac{M_t l}{G I_o} \\ \theta\ [^\circ] &= \frac{180^\circ}{\pi}\theta = \frac{180}{\pi}\frac{M_t l}{G I_o} \end{aligned}}$$

II glavna j-na uvijanja

* Kružni poprečni presek :



$$R = \frac{d}{2}$$

$$I_o = \frac{1}{2}R^4\pi = \frac{1}{32}d^4\pi \approx \frac{1}{10}d^4$$

$$\Rightarrow W_o = \frac{1}{2}R^3\pi = \frac{1}{16}d^3\pi \approx \frac{1}{5}d^3$$

$$\Rightarrow \tau_{max} = \frac{2M_t}{R^3\pi} = \frac{16M_t}{d^3\pi} \approx \frac{5M_t}{d^3}$$

$$\tau_{max} \le \tau_{dt} \Rightarrow \left\{ d \ge \sqrt[3]{\frac{16M_t}{\pi\tau_{dt}}} \approx \sqrt[3]{\frac{\overset{[Ncm]}{5M_t}}{\underset{[N/cm^2]}{\tau_{dt}}}}\ [cm] \right\}$$

ZADATAK BR. 9

Čelična konzola, kružnog poprečnog preseka, opterećena je na slobodnom kraju momentom uvijanja $M_t = 800\,[Nm]$.
Dimenzionisati konzolu ako je dozvoljeni napon na uvijanje $200\ N/cm^2$.
Odrediti ugao uvijanja za raspon konzole $l = 2\ m$. $G = 8 \cdot 10^5\ N/cm^2$.

$$\tau_{max} = \frac{M_t \cdot R}{I_o} = \frac{M_t}{W_o} \leq \tau_{dt} \Rightarrow W_o \geq \frac{M_t}{\tau_{dt}}$$

$$W_o = \frac{I_o}{R} = \frac{1}{2} R^3 \pi = \frac{1}{16} d^3 \pi \geq \frac{M_t}{\tau_{dt}} \Rightarrow d \geq \sqrt[3]{\frac{16\, M_t}{\pi\, \tau_{dt}}} = 12{,}68\ cm$$

usvaja se $d = 13\ cm$

$$\tau_{max} = \frac{M_t}{W_o} = \frac{16\, M_t}{d^3 \pi} = 185{,}45\ \frac{N}{cm^2} < \tau_{dt} = 200\ \frac{N}{cm^2}$$

$$\theta = \frac{M_t \cdot l}{G\, I_o} = \frac{l}{d}\,\frac{\tau_{max}}{G} = 0{,}00713692\ [rad]$$

$$\theta^\circ = \frac{180^\circ}{\pi}\theta = 0{,}41^\circ = 0^\circ\, 24'\, 31''$$

ZADATAK BR. 10

Koliki je najveći tangencijalni napon grede kružnog poprečnog preseka, prečnika 110 mm, ako je izložena momentu uvijanja $M_t = 1{,}4\ kNm$?

$$\tau_{max} = \frac{M_t}{W_o} = \frac{M_t}{\frac{1}{16} d^3 \pi} = \frac{16\, M_t}{d^3 \pi} = \frac{16 \cdot 1{,}4 \cdot 10^3 \cdot 10^2}{11^3 \cdot \pi} \approx 535{,}7\ \frac{N}{cm^2}$$

ZADATAK BR. 11

Greda kružnog poprečnog preseka, prečnika 100 mm, ima tangencijalni napon $350\ N/cm^2$ u tački poprečnog preseka na rastojanju 10 mm od središta.
Koliki je moment uvijanja grede? $\left[d = 10\ cm,\ R = \frac{d}{2} = 5\,cm\right]$

$$\tau_{max} = \frac{2 M_t}{R^3 \pi} \Rightarrow M_t = \frac{1}{2} \tau_{max} R^3 \pi$$

$$\tau = \frac{M_t\, r}{I_o}, \quad \tau_{max} = \frac{M_t\, R}{I_o} \Rightarrow \frac{\tau}{\tau_{max}} = \frac{r}{R} \Rightarrow \tau_{max} = 1750\ \frac{N}{cm^2}$$

$$\Rightarrow M_t = 3{,}5\ kNm$$

ZADATAK BR. 12

Vratilo kružnog poprečnog preseka, prečnika 80 mm, prenosi snagu 300 Nm/s pri 240 obr./min.
Odrediti najveći tangencijalni napon i ugao uvijanja vratila ako je njegova dužina 5 m, a modul klizanja 800 kN/cm².

$P = 300 \left[\frac{Nm}{s}\right]$, $n = 240 \left[\frac{ob}{min}\right]$, $d = 80mm = 8\,cm$

$l = 5\,[m]$, $G = 800\,[kN/cm^2]$

$$P = M_t \cdot \omega = M_t \cdot \frac{\pi n}{30} \Rightarrow M_t = \frac{30P}{\pi n} = \frac{30 \cdot 300}{\pi \cdot 240} = 12\ Nm$$

$$\tau_{max} = \frac{16 M_t}{d^3 \pi} = \frac{16 \cdot 1200}{8^3 \pi} = 11{,}94 \frac{N}{cm^2}$$

$$\theta = \frac{M_t l}{G I_o} = \frac{M_t \cdot l}{G \frac{1}{32} d^4 \pi} = \frac{32 M_t l}{G d^4 \pi} = \frac{32 \cdot 12 \cdot 10^2 \cdot 500}{800 \cdot 10^3 \cdot 8^4 \pi} = 0{,}001865\ rad$$

ZADATAK BR. 13

Vratilo dužine 6m, kružnog poprečnog preseka, R = 100 mm, uvija se za 4°. Koliki je najveći tangencijalni napon ako je materijal čelik ($G = 8 \cdot 10^5\ N/cm^2$)?

$l = 6m = 600\,cm$

$R = 100\,mm = 10\,cm \Rightarrow d = 5\,cm$

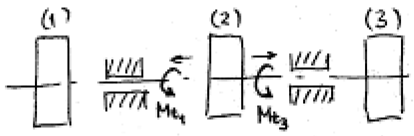
$\theta = 4°$

$\tau_{max} = ?$

$$\theta = \frac{180}{\pi} \frac{l}{d} \frac{\tau_{max}}{G} \Rightarrow \tau_{max} = \frac{\theta \pi d G}{180 \cdot l} = \frac{4 \cdot \pi \cdot 5 \cdot 8 \cdot 10^5}{180 \cdot 600} = 465{,}42 \frac{N}{cm^2}$$

ZADATAK BR. 14

Transmisiono vratilo prima preko kaišnika (2) snagu 7500 W, a odaje preko kaišnika (1) snagu $P_1 = 3000$ W i preko kaišnika (3) snagu $P_3 = 4500$ W.
Dimenzionisati vratilo kružnog poprečnog preseka ako je $\tau_{dt} = 120\ N/cm^2$ i $n = 175$ $\%_{min}$.



$$M_{t_i} = \frac{P_i}{\omega} = \frac{P}{\frac{\pi n}{30}} = \frac{30 P_i}{\pi \cdot n}$$

$$M_{t_1} = \frac{30 P_1}{\pi n} = \frac{30 \cdot 3000}{\pi \cdot 175} = 164\ Nm = 16400\ Ncm$$

$$M_{t_2} = 41\,000\ Ncm$$

$$M_{t_3} = \frac{30 P_3}{\pi \cdot n} = M_{t_2} - M_{t_1} = 24\,600\ Ncm$$

$$d \geq \sqrt[3]{\frac{5 M_{t\,max_{1,3}}}{\tau_{dt}}} = \sqrt[3]{\frac{5 \cdot M_{t_3}}{\tau_{dt}}} = 10{,}07\ cm \text{ , usvaja se } d = 11\ cm.$$

ZADATAK BR. 15

Odrediti τ_{max} grede prstenastog poprečnog preseka, odnosa $\psi = \frac{r}{R} = \frac{1}{2}$, ako je $R = 100$ mm, a $M_t = 3$ kNm.

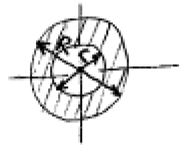
$$\tau_{max} = \frac{M_t}{W_o}$$

$$W_o = \frac{I_o}{R} = \frac{\frac{\pi R^4}{2} - \frac{\pi r^4}{2}}{R} = \frac{\pi R^3}{2}(1 - \psi^4)$$

$$\tau_{max} = \frac{2 M_t}{\pi R^3 (1 - \psi^4)} \approx 200\ N/cm^2$$

ZADATAK BR. 16

Dimenzionisati gredu kružno-prstenastog poprečnog preseka, odnosa $\psi = \frac{r}{R} = \frac{1}{2}$, ako prima moment uvijanja $M_t = 2\ kNm$ i ukoliko je $\tau_{dt} = 320\ N/cm^2$.

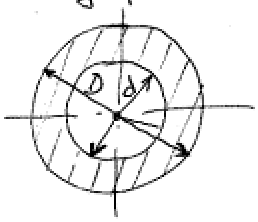


$$\tau_{max} = \frac{M_t}{W_o} = \frac{M_t}{\frac{\pi R^3}{2}(1-\psi^4)} = \frac{2M_t}{\pi R^3(1-\psi^4)} \leq \tau_{dt}$$

$$\Rightarrow R \geq \sqrt[3]{\frac{2M_t}{\pi \tau_{dt}(1-\psi^4)}} = 7{,}4\ cm \quad ; \quad \text{usvaja se } R = 8\ cm$$

ZADATAK BR. 17

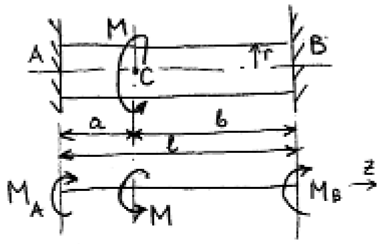
Transmisiono vratilo predaje snagu $15\ kNm/s$ pri $100\ ^{ob}/_{min}$. Koliki je τ_{tmax} ako je vratilo kružno-prstenastog preseka $\frac{D}{d} = \frac{120\ mm}{70\ mm}$?



$$\tau_{max} = \frac{M_t}{W_0} = \frac{\frac{30P}{\pi n}}{\frac{\pi D^3}{32}(1-\psi^4)} = 470\ N/cm^2$$

ZADATAK BR. 18

Štap kružnog preseka, poluprečnika r, uklješten je između dva zida na razmaku ℓ. Štap je opterećen obrtnim momentom M. Odrediti momente ukleštenja M_A i M_B i ugao uvijanja između preseka A i C.



Odstranimo ukljěstenja i zamenimo njihov uticaj na štap momentima M_A i M_B. Uslov ravnoteže je:

$$\sum \overset{\curvearrowright +}{M_z} = 0 \quad : \quad M_A - M + M_B = 0$$

Postoje dve nepoznate pa je zadatak jedanput statički neodređen.

Geometrijski uslov je da je ugao uvijanja između krajnjih preseka jednak nuli:

$$\theta_{A-B} = 0 \Rightarrow \frac{1}{GI_o}\int_0^\ell M_t\,dz = 0 \rightarrow \int_0^\ell M_t\,dz = 0$$

$$\int_0^a M_{t_1}dz + \int_0^b M_{t_2}dz = M_A \cdot a + (M_A - M)\cdot b = 0 \Rightarrow M_A = \frac{M\cdot b}{\ell}, \quad M_B = \frac{Ma}{\ell}$$

$$\theta_{A-C} = \frac{1}{GI_o}\int_0^a M_t\,dz = \frac{M_A\cdot a}{GI_o} = \frac{Mab}{GI_o\ell}$$

3. SAVIJANJE (FLEKSIJA)

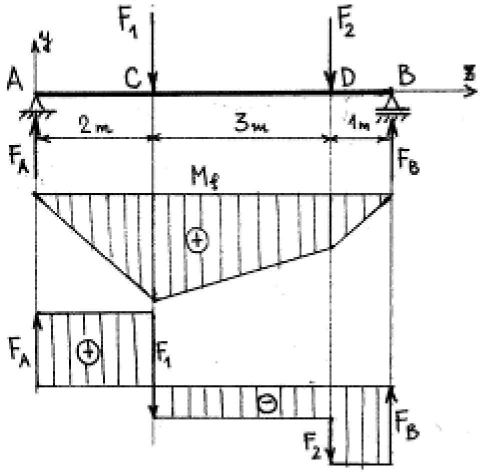
$$\sigma_x = \frac{M_f}{I_x}\cdot y \quad \text{– I glavna j-na savijanja}$$

$$k = \frac{1}{R_k} = \frac{M_f}{EI_x} \quad \text{– II glavna j-na savijanja}$$

$$\tau = \frac{T S_x^{ods.}}{I_x \cdot b(y)} \quad \text{– III glavna j-na savijanja}$$

ZADATAK BR. 19

Prosta greda AB, raspona $l = 6\,m$, opterećena je teretima $F_1 = 40$ kN i $F_2 = 20$ kN. Dimenzionisati čeličnu gredu kružnog poprečnog preseka ako je dozvoljeni napon na savijanje $\sigma_{df} = 120$ MPa.



1° $\sum Y = 0$; $F_A - F_1 - F_2 + F_B = 0$

$F_A + F_B = F_1 + F_2 = 60$

$\sum \vec{M}_A = 0$; $F_1 \cdot 2 + F_2 \cdot 5 - F_B \cdot 6 = 0$

$$F_B = \frac{2F_1 + 5F_2}{6} = \frac{2\cdot 40 + 5\cdot 20}{6}$$

$F_B = 30$ kN

$F_A = 30$ kN

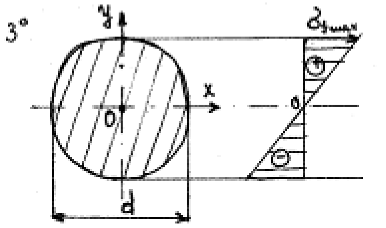
2° $M_A = M_B = 0$

$M_C = F_A \cdot 2 = 60$ kNm

$M_D = F_B \cdot 1 = 30$ kNm

$M_{f_{max}} = 60$ kNm $= 60\cdot 10^5$ Ncm

3°



$$\sigma_{f_{max}} = \frac{M_{f_{max}}}{I_x}\cdot y_{max} \le \sigma_{df} \;;$$

$$W_x = \frac{I_x}{y_{max}} \Rightarrow \sigma_{f_{max}} = \frac{M_{f_{max}}}{W_x} \le \sigma_{df} \;; \Rightarrow W_x \ge \frac{M_{f_{max}}}{\sigma_{df}} = \frac{60\cdot 10^5}{120\cdot 10^6 \cdot 10^{-4}} = 500\,cm^3$$

$$y_{max} = \frac{d}{2} \;;\; I_x = \frac{\pi d^4}{64} \;;\; W_x = \frac{\pi d^3}{32} \;;$$

$$d \ge \sqrt[3]{\frac{32 W_x}{\pi}} \Rightarrow d \ge 17{,}2\,cm \;;$$ USVAJA SE $d = 180$ [mm]

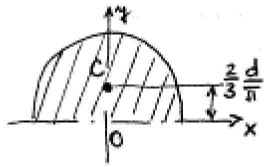
4° Provera:

$\tau_{df} = 0{,}8\,\sigma_{df} \Rightarrow \tau_{df} = 96\ MPa\ ;\quad \tau_{max} \leq \tau_{df}$

$$\sigma_{f_{max}} = \frac{M_{f_{max}}}{W_x} = \frac{M_f}{\frac{\pi d^3}{32}} = \frac{32 \cdot M_f}{\pi d^3} = \frac{32 \cdot 60 \cdot 10^5}{\pi \cdot 18^3} = 10479{,}33\ \frac{N}{cm^2} = 10{,}48 \cdot 10^7\ [Pa]$$

$\sigma_{f_{max}} = 104{,}8\ MPa$

$$\tau_{max} = \frac{T_{max} \cdot S_x'}{I_x \cdot b}\ ;\quad I_x = \frac{\pi d^4}{64}\ ,\quad b = d\ ,\quad T_{max} = 30\ kN = 30 \cdot 10^3\ N$$



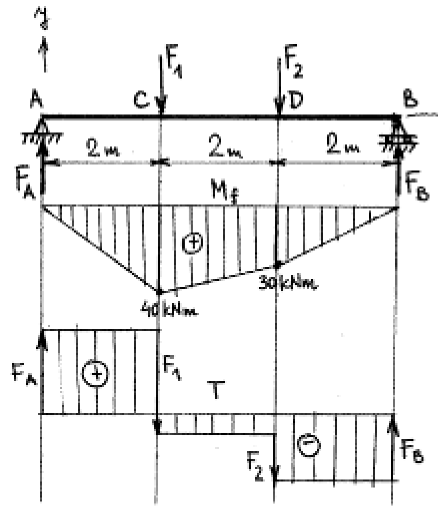
$$S_x' = S_x^{1/2} = \frac{d^3}{12} = \frac{18^3}{12} = 486\ cm^3$$

$I_x = 5152{,}9973\ cm^4\ ;\quad b = d = 18\ cm$

$$\tau_{max} = 157{,}19\ \frac{N}{cm^2} = 157{,}19 \cdot 10^4\ \frac{N}{m^2} = 1{,}5719\ MPa \leq \tau_{df}$$

ZADATAK BR. 20

Prosta greda AB, raspona $l = 6m$, opterećena je silama $F_1 = 25$ kN i $F_2 = 10$ kN. Dimenzionisati čeličnu gredu pravougaonog poprečnog preseka, odnosa strana $b : h = 1 : 2$, ako je $\sigma_{df} = 120$ [MPa].



$F_1 = 25$ kN, $F_2 = 10$ kN, $\frac{b}{h} = \frac{1}{2}$, $\sigma_{df} = 120$ MPa

1° $\Sigma Y = 0$:

$F_A - F_1 - F_2 + F_B = 0 \Rightarrow F_A + F_B = 35$

$\Sigma \overset{+}{M} = 0$:

$F_1 \cdot 2 + F_2 \cdot 4 - F_B \cdot 6 = 0 \Rightarrow F_B = \frac{2F_1 + 4F_2}{6}$

$F_B = 15$ kN ; $F_A = 20$ kN

2° $M_A = M_B = 0$

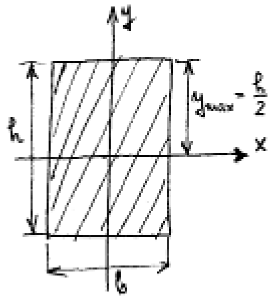
$M_C = F_A \cdot 2 = 40$ kNm

$M_D = F_B \cdot 2 = 30$ kNm

$M_{fmax} = 40 \text{ kNm} = 40 \cdot 10^5$ Ncm

$\sigma_{df} = 120 \text{ MPa} = 120 \cdot 10^6 \frac{N}{m^2} = 120 \cdot 10^2 \frac{N}{cm^2}$

$\sigma_{fmax} = \frac{M_{fmax}}{W_x} \le \sigma_{df} \Rightarrow W_x \ge \frac{M_{fmax}}{\sigma_{df}} \Rightarrow W_x \ge \frac{1000}{3}$ [cm³]



$W_x = \frac{I_x}{y_{max}} = \frac{\frac{bh^3}{12}}{\frac{h}{2}} = \frac{bh^2}{6}$; $\frac{b}{h} = \frac{1}{2} \Rightarrow h = 2b$

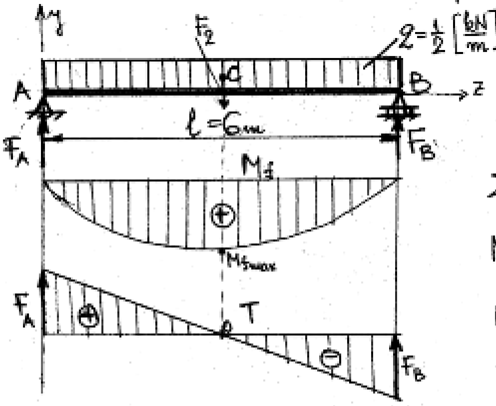
$W_x = \frac{b(2b)^2}{6} = \frac{2}{3}b^3$

$\frac{2}{\cancel{3}}b^3 \ge \frac{1000}{\cancel{3}} \Rightarrow b^3 \ge 500 \Rightarrow b \ge 7,937$ cm

USVAJA SE : $b = 8$ [cm] $\Rightarrow h = 16$ [cm]

ZADATAK BR. 21

Dimenzionisati čeličnu gredu kvadratnog poprečnog preseka, dužine $l = 6\,m$, ako je opterećena jednolikim opterećenjem $q = \frac{1}{2}\,[kN/m]$. Dozvoljeni napon iznosi $\sigma_{df} = 1000\ N/cm^2$.

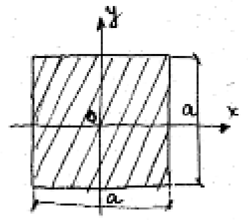


$$F_q = q \cdot l = \frac{1}{2} \cdot 6 = 3\ [kN]$$

$$\Sigma Y = 0 \Rightarrow F_A = F_B = \frac{F_q}{2} = 1{,}5\,[kN]$$

$$M_{fmax} = M_C = F_A \cdot \frac{l}{2} - q\frac{l}{2} \cdot \frac{l}{4} = 2{,}25\ kNm$$

$$M_{fmax} = 2{,}25 \cdot 10^5\ Ncm$$



$$\sigma_{fmax} = \frac{M_{fmax}}{W_x} \leq \sigma_{df}$$

$$W_x = \frac{I_x}{y_{max}} = \frac{\frac{a^4}{12}}{\frac{a}{2}} = \frac{a^3}{6}$$

$$\frac{a^3}{6} \geq \frac{M_{fmax}}{\sigma_{df}} \Rightarrow a \geq \sqrt[3]{6\,\frac{2.25 \cdot 10^5}{10^3}} \geq 11{,}52\,cm$$

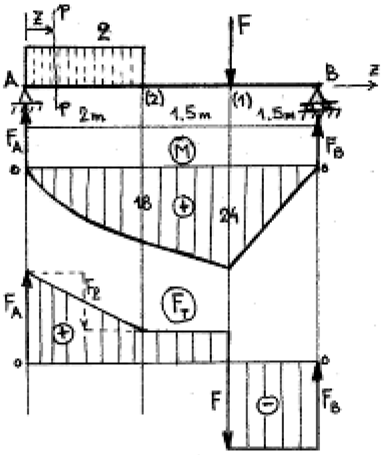
USVAJA SE: $\underline{a = 12\ cm}$

ZADATAK BR. 22

Prosta greda opterećena je koncentrisanom silom $F = 20\ kN$ i jednako raspodeljenim kontinualnim opterećenjem $q = 5\ kN/m$. Dimenzionisati prostu gredu ako je $\sigma_{df} = 12\ kN/cm^2$ i ako je ona:

a) kružnog poprečnog preseka;

b) pravougaonog poprečnog preseka odnosa stranica $b/h = 2/3$;



$$\Sigma Y = 0: \quad F_A + F_B - 2q - F = 0$$
$$F_A + F_B = 30$$
$$\Sigma M_B = 0: \quad 5F_A - 2q \cdot 4 - 1{,}5F = 0$$
$$5F_A = 40 + 30 \Rightarrow \boxed{F_A = 14\ kN}$$
$$\boxed{F_B = 16\ kN}$$

$$M_1 = F_B \cdot 1{,}5 = 16 \cdot 1{,}5 = 24\ kNm = M_{f\,max}$$
$$M_2 = F_B \cdot 3 - F \cdot 1{,}5 = 48 - 30 = 18\ kNm$$
$$M_{f(p-p)} = F_A \cdot z - q \cdot z \cdot \frac{z}{2} = F_A \cdot z - \frac{1}{2} q z^2 \Big|_{0 \le z \le 2m}$$
$$\frac{dM_f}{dz} = F_A - qz = 0, \quad z = \frac{F_A}{q} = \frac{14}{5} = 2{,}8\ m$$

a) $W_x = \frac{\pi d^3}{32}$, $\sigma_{f\,max} = \frac{M_{f\,max}}{W_x} \le \sigma_{df} \Rightarrow W_x \ge \frac{M_{f\,max}}{\sigma_{df}}$, $\frac{\pi d^3}{32} \ge \frac{24 \cdot 100}{12}$

$d \ge \sqrt[3]{\frac{200 \cdot 32}{\pi}}$, $d \ge 12{,}68\ cm$; usvaja se $d = 13\ cm$

b) $W_x = \frac{bh^2}{6}$, $W_x \ge \frac{M_{f\,max}}{\sigma_{df}}$ $\therefore$ $\frac{bh^2}{6} \ge \frac{24 \cdot 100}{12} \Rightarrow \frac{2}{3}h \cdot \frac{h^2}{6} \ge \frac{24 \cdot 100}{12}$

$h^3 \ge 1800$, $h \ge \sqrt[3]{1800} \Rightarrow h \ge 12{,}15\ cm$

$b = \frac{2}{3}h = \frac{2}{3}12{,}15 = 8{,}11\ cm$

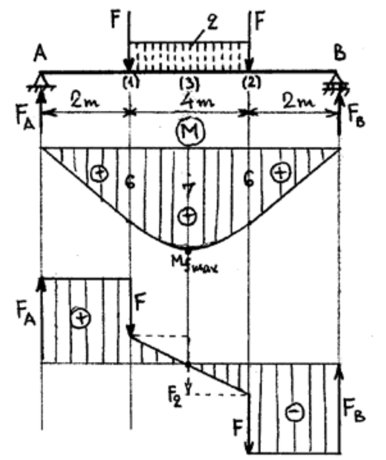
ZADATAK BR. 23

Greda $\overline{AB}$, raspona 8m, poprečnog preseka predstavljenog na slici, opterećena je teretima F = 2 kN i q = 0,5 kN/m.

a) Odrediti najveće normalne napone (ivične) u opasnom preseku grede i nacrtati dijagram normalnog napona.

b) U preseku grede na udaljenju 1,5 m od levog oslonca odrediti normalni i tangencijalni napon u tačkama preseka na rastojanju 10 cm od neutralne ose.

c) U tim tačkama (zadatak pod b) odrediti veličinu glavnih napona.



$\Sigma Y = 0: \; F_A + F_B = 2F + 4q$

$\Sigma M_B = 0: \; F_A \cdot 8 - F \cdot 6 - 4q \cdot 4 - F \cdot 2 = 0$

Simetrija je potpuna, pa je:

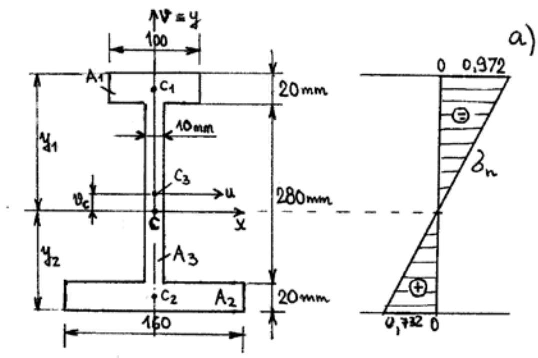
$F_A = F_B = F + 2q = 2 + 2 \cdot 0,5 \Rightarrow F_A = F_B = 3\ kN$

$M_1 = M_2 = F_A \cdot 2 = 3 \cdot 2 \Rightarrow M_1 = M_2 = 6\ kNm$

$M_3 = F_A \cdot 4 - 2F - 2q \cdot 1 = 3 \cdot 4 - 2 \cdot 2 - 2 \cdot 0,5 \cdot 1 = 12 - 4 - 1$

$M_3 = 7\ kNm = M_{fmax}$

(opasan presek)



a) $A = A_1 + A_2 + A_3 = 10 \cdot 2 + 16 \cdot 2 + 28 \cdot 1$

$A = 80\ cm^2$

$v_c = \dfrac{\Sigma A_i v_i}{\Sigma A_i} = \dfrac{20 \cdot 15 - 32 \cdot 15}{80} = -\dfrac{9}{4}\ cm$

$v_c = -2,25\ cm$

$y_1 = 16 + v_c = 16 + 2,25 = 18,25\ cm$

$y_2 = 16 - v_c = 16 - 2,25 = 13,75\ cm$

$$I_x = \left(\frac{2^3 \cdot 10}{12} + 2 \cdot 10 \cdot 17,25^2\right) + \left(\frac{2^3 \cdot 16}{12} + 2 \cdot 16 \cdot 12,75^2\right) + \left(\frac{1 \cdot 28^3}{12} + 1 \cdot 28 \cdot 2,25^2\right)$$

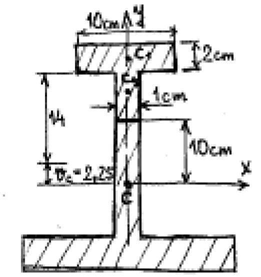
$I_x = 13\,141,667\ cm^4$

$W_{x_1} = \dfrac{I_x}{y_1} = \dfrac{13141,667}{18,25} = 720,1\ cm^3 \; ; \quad W_{x_2} = \dfrac{I_x}{y_2} = \dfrac{13141,667}{13,75} = 955,76\ cm^3$

$$\sigma_{max} = \frac{M_{fmax}}{W_{x_{min}}} = \frac{M_{fmax}}{W_{x_1}} = \frac{7 \cdot 100}{720,1} = 0,972\ \frac{kN}{cm^2}$$

$$\sigma_{min} = \frac{M_{fmax}}{W_{x_{max}}} = \frac{M_{fmax}}{W_{x_2}} = \frac{7 \cdot 100}{955,76} = 0,732\ \frac{kN}{cm^2}$$

b) $M(z=1{,}5) = F_A \cdot 1{,}5 = 3 \cdot 1{,}5 = 4{,}5\ \text{kNm}\ ,\quad F_T = 3\ \text{kN}$



$$\sigma = \frac{M}{I_x} \cdot y = -\frac{4{,}5 \cdot 100}{13141{,}667} \cdot 10 = -0{,}343\ \frac{\text{kN}}{\text{cm}^2}$$

$$\tau = \frac{F_T \cdot S_x'}{I_x \cdot \xi}\quad ;\quad \xi = 1\ \text{cm}$$

$$S_x' = 20 \cdot 17{,}25 + 1 \cdot (2{,}25 + 14 - 10) \cdot \left[10 + \frac{1}{2}(2{,}25 + 14 - 10)\right]$$

$$S_x' = 427\ \text{cm}^3$$

$$\tau = \frac{3 \cdot 427}{13141{,}667 \cdot 1} = 0{,}0975\ \frac{\text{kN}}{\text{cm}^2}$$

c) $$\sigma_{1/2} = \frac{\sigma}{2} \pm \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2} = \frac{1}{2}\left(-0{,}343 \pm \sqrt{0{,}343^2 + 4 \cdot 0{,}0975^2}\right)$$

$$\sigma_{1/2} = \frac{1}{2}(-0{,}343 \pm 0{,}395) \Rightarrow \sigma_1 = \frac{1}{2}(-0{,}343 - 0{,}395) = -0{,}369\ \frac{\text{kN}}{\text{cm}^2}$$

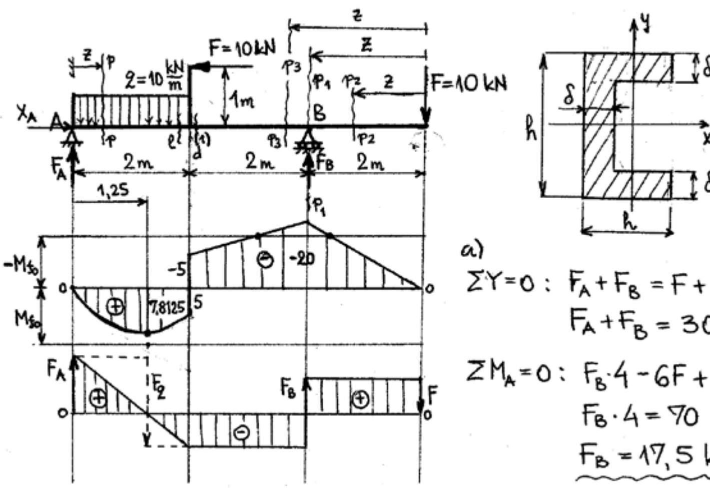
$$\sigma_2 = \frac{1}{2}(-0{,}343 + 0{,}395) = 0{,}026\ \frac{\text{kN}}{\text{cm}^2}$$

ZADATAK BR. 24

a) Za nosač na slici odrediti otpore oslonaca i nacrtati statičke dijagrame.

b) Dimenzionisati nosač prema najvećem momentu savijanja ako je dozvoljeni napon na savijanje $\sigma_{df} = 10\ kN/cm^2$.

c) Izračunati normalni i tangencijalni napon u preseku $p_1 - p_1$, a u tačkama na udaljenju $h/4$ od gornje ivice profila, visine h.



$$\psi = \frac{\delta}{h} = \frac{1}{4}$$

a)

$$\Sigma Y = 0: \ F_A + F_B = F + 2q = 30$$

$$F_A + F_B = 30$$

$$\Sigma M_A = 0: \ F_B \cdot 4 - 6F + 1 \cdot F - 1 \cdot 2 \cdot 2 = 0$$

$$F_B \cdot 4 = 70$$

$$F_B = 17,5\ kN \ ; \ F_A = 12,5\ kN$$

$$\Sigma X_A = 0 : \ X_A = F = 10\ kN$$

$$M_B = -2F = -20\ kNm \ ; \ M_1^l = -4F + 2F_B + 1 \cdot F = 5\ kNm$$

$$M_1^d = -4F + 2F_B = -5\ kNm$$

$$M_{p-p} = F_A \cdot z - qz \cdot \frac{1}{2} z = F_A \cdot z - \frac{1}{2} q z^2$$

$$\frac{dM_{p-p}}{dz} = F_A - qz = 0 \Rightarrow z = \frac{F_A}{q} = \frac{12,5}{10} = 1,25\ m \ ;$$

$$M_{(z=1,25)} = 12,5 \cdot 1,25 - 10 \cdot \frac{1,25^2}{2} = 7,8125\ kNm$$

b)

$$I_x = \frac{h^4}{12} - \frac{(h-\delta)(h-2\delta)^3}{12} = \frac{h^4}{12}\left[1-(1-\psi)(1-2\psi)^3\right] = \frac{29}{12 \cdot 32} h^4$$

$$W_x = \frac{I_x}{y_{max}} = \frac{29 \cdot h^4}{12 \cdot 32 \cdot \frac{h}{2}} = \frac{29}{6 \cdot 32} h^3$$

$$M_{f\,max} = |-20|\ kNm$$

$$\sigma_f = \frac{M_{f\,max}}{W_x} \leq \sigma_{df} \Rightarrow W_x \geq \frac{M_{f\,max}}{\sigma_{df}} \ , \ \frac{29 h^3}{6 \cdot 32} \geq \frac{20 \cdot 100}{10}$$

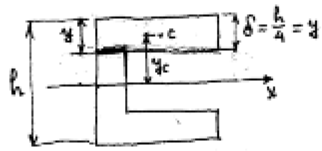
$$h \geq \sqrt[3]{\frac{6 \cdot 32 \cdot 200}{29}} \ , \ h \geq 10,98\ cm$$

USVAJA SE $h = 11\ cm$

$$W_x = \frac{29 \cdot h^3}{6 \cdot 32} = \frac{29 \cdot 11^3}{6 \cdot 32} = 201,036\ cm^3$$

c) $M_{f_{P_1-P_1}} = -20\ kNm$, $\delta/h = 1/4$, $\delta = \frac{h}{4} = \frac{11}{4}\ cm$; $y = \frac{h}{4} = \frac{11}{4}\ cm$

$$\sigma_{f_B} = \frac{M_{f_B}}{I_x} \cdot y = \frac{20 \cdot 100}{\frac{29 \cdot 11^4}{12 \cdot 32}} \cdot \frac{11}{4} = 4{,}97\ \frac{kN}{cm^2}$$



$F_{T_B} = 10\ kN$;

$$S_x' = A \cdot y_c = h\delta\left(\frac{h}{2} - \frac{\delta}{2}\right) = h\frac{h}{4}\left(\frac{h}{2} - \frac{h}{8}\right) = \frac{3}{32}h^3$$

$\xi_1 = h$, $\xi_2 = \frac{h}{4} = \delta$

$$\tau_{B_1} = \frac{F_{TB} \cdot S_x'}{I_x\ \xi_1} = \frac{10 \cdot \frac{3 \cdot 11^3}{32}}{\frac{29 \cdot 11^4}{12 \cdot 32} \cdot 11} = \frac{10 \cdot 3 \cdot 12}{29 \cdot 11 \cdot 11} = 0{,}1025\ \frac{kN}{cm^2}$$

$$\tau_{B_2} = \frac{F_{T_B} \cdot S_x'}{I_x\ \xi_2} = 0{,}41\ \frac{kN}{cm^2}$$